Практика

„Квантовая механика"

студентки 4-го курса

группа А.А-43

Самиховой Инны

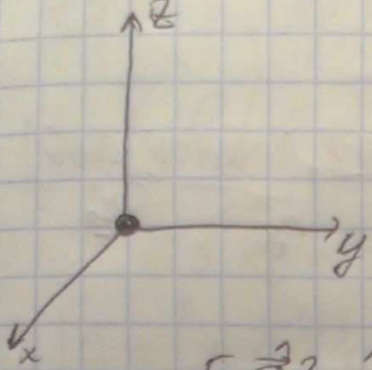
Спин

5.09.2012

$\varphi(\vec r, s) = \psi(\vec r)\,\chi(s)$

$\hat{\vec S} = (\hat S_x, \hat S_y, \hat S_z)$

$\hat{\vec S}^2 = \hat S_x^2 + \hat S_y^2 + \hat S_z^2$



$\hat S \chi(s) = s(s+1)\chi(s)$;

$[\hat{\vec S}^2, \hat S_z] = 0$;

$\hat S_z \chi(s) = s_z \chi(s)$

$(2s+1)$

$S = 1/2$

$[\hat S_i, \hat S_k] = i\,\varepsilon_{ikj} S_j$

$\frac{1}{2}\hat{\vec \sigma} = \hat{\vec S}$

$\hat\sigma_x = 2\hat S_x$

$\hat S = 1/2$

$\hat\sigma_y = 2\hat S_y$

$S_z = \pm 1/2$

$\hat\sigma_z = 2\hat S_z$

$(\sigma_z)_{\sigma\sigma} = \sigma\,\delta_{\sigma\sigma}$

$\hat S_z = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$

Доказ. $\rightarrow$ $(S_x)_{\sigma,\sigma-1} = \frac{1}{2}\sqrt{(s+\sigma)(s-\sigma+1)} = (S_x)_{\sigma-1,\sigma}$

$(S_y)_{\sigma,\sigma-1} = -\frac{i}{2}\sqrt{(s+\sigma)(s-\sigma+1)} = -(S_y)_{\sigma-1,\sigma}$

$(S_x)_{11} = (S_x)_{22} = (S_y)_{11} = (S_y)_{22}$

$(S_x)_{12} = (S_x)_{21} = 1/2$

$S_x = \frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$

$(S_y)_{12} = -(S_y)_{21} = -i/2$

Матрицы Паули

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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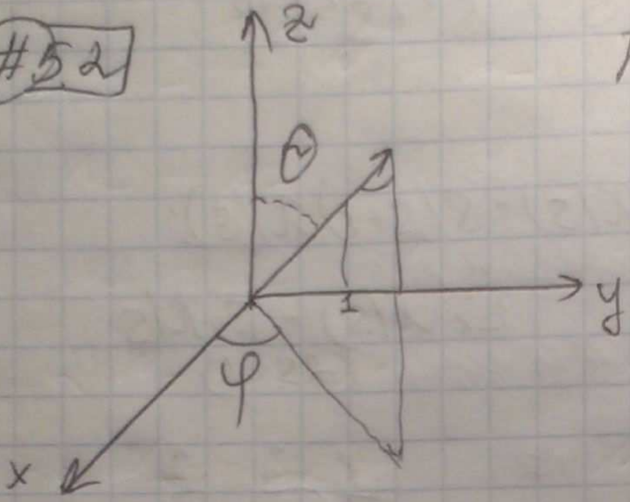
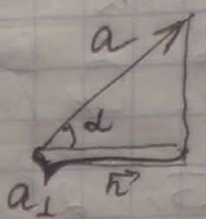


$$\vec{n} = (\underbrace{\sin\theta\cos\varphi}_{n_x}, \underbrace{\sin\theta\sin\varphi}_{n_y}, \underbrace{\cos\theta}_{n_z})$$



$$\hat{\vec{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$$

$$\hat{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \quad \hat{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{a}\cdot\vec{n} = a \cdot n \cos\alpha = a_\perp$$

$$\vec{n}\cdot\hat{\vec{S}} = \frac{1}{2}\sin\theta\cos\varphi\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{1}{2}\sin\theta\sin\varphi\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} +$$

$$+ \frac{1}{2}\cos\theta\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 1/2\sin\theta\cos\varphi \\ 1/2\sin\theta\cos\varphi & 0 \end{pmatrix} +$$

$$+ \begin{pmatrix} 0 & -i/2\sin\theta\sin\varphi \\ i/2\sin\theta\sin\varphi & 0 \end{pmatrix} + \begin{pmatrix} 1/2\cos\theta & 0 \\ 0 & -1/2\cos\theta \end{pmatrix} =$$

$$= \begin{pmatrix} 1/2\cos\theta & \sin\theta(1/2\cos\varphi - i/2\sin\varphi) \\ \sin\theta(1/2\cos\varphi + i/2\sin\varphi) & -1/2\cos\theta \end{pmatrix}$$

$$\vec{n}\cdot\vec{S} = \underbrace{\frac{1}{2}\begin{pmatrix}\cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta\end{pmatrix}}_{} = \hat{S}_n$$

$$\langle f \rangle = \int dq\, \psi^*(q)\hat{f}\psi(q) = \sum_i |a_i|^2 f_i$$

$$\psi(q) = \sum_i a_i \psi_i(q)$$

$$\langle f \rangle_i = f_i \qquad \psi_a(q) = \langle q|a\rangle$$

$$|a\rangle^+ = \langle a|$$

$$\psi_{S_z} = \alpha\psi_{1/2} + \beta\psi_{-1/2} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix}$$

$$\begin{pmatrix}1\\0\end{pmatrix} = |S_z\rangle \qquad \begin{pmatrix}0\\1\end{pmatrix} = |S_z\rangle$$

$$S_z = 1/2 \qquad S_z = -1/2$$

$$\langle S_n \rangle_{S_z = 1/2} = \langle S_z|S_n|S_z\rangle = (1\,0)\,\hat{S}_n\begin{pmatrix}1\\0\end{pmatrix}$$

$$(\hat{A}\hat{B})_{mk} = \sum_k a_{mk}$$

$$\langle \hat{S}_n \rangle_{S_z = -1/2} = -\frac{1}{2}\cos\theta$$

$$\langle \hat{S}_n \rangle_{S_z = 1/2} = \frac{1}{2}\cos\theta$$

$$\langle \hat{S}_n \rangle_{S_z} = \hat{S}_z \cos\theta$$

$$W_+ \left(\hat{S}_n = \frac{1}{2}\right)$$

$$W_- \left(\hat{S}_n = -\frac{1}{2}\right) \qquad \hat{S}_z = \frac{1}{2}$$

$$\begin{cases} \langle \hat{S}_n \rangle_{1/2} = \frac{1}{2}\cos\Theta = \frac{1}{2} W_+ + (-1/2) W_- \\ W_+ + W_- = 1 \end{cases}$$

$$\begin{cases} \cancel{1/2}\cos = \cancel{1/2}(W_+ - W_-) \\ 1 = W_+ + W_- \end{cases}$$

$$\pm\begin{cases} W_+ - W_- = \cos\Theta \\ W_+ + W_- = 1 \end{cases}$$

$$\begin{cases} 2W_+ = \cos\Theta + 1 \\ -2W_- = \cos\Theta - 1 \end{cases} \qquad \begin{cases} W_+ = \frac{\cos\Theta + 1}{2} > 0 \\ W_- = -\frac{\cos\Theta - 1}{2} \end{cases}$$

$W_+ = \cos^2 \Theta/2$

$W_- = \sin^2 \Theta/2$ $\qquad \boxed{S_z = -1/2}$ $-?$

12.09.2012

(7) Частицу со спином $S=1$. Вычислить вероятность проекции спина на выдел. напр.

$S=1$ $\quad W_1 = W_{++}$ $\quad (|1\,1\rangle) - ?$

$W_2 = W_{--}$ $\quad (|1\,-1\rangle) - ?$

$$W_0 = \begin{cases} W_{+-} & (|1\,0\rangle) - ? \\ + & \\ W_{-+} & (|0\,1\rangle) - ? \end{cases}$$

$S = 1/2$ $\quad W_0 = W_{+-} + W_{-+} - ?$

$W_+ = \cos^2 \Theta/2$

$W_- = \sin^2 \Theta/2$

$W_{++} = \cos^4 \theta/2$

$W_{--} = \sin^4 \theta/2$

$W_{+-} = W_{-+} = \cos^2 \theta/2 \sin^2 \theta/2$

$W_0 = 2\cos^2 \theta/2 \sin^2 \theta/2 = \left\{\sin^2 \theta\right\}$

$W = W_{++} + W_{--} + W_0 = \underbrace{\cos^4 \theta/2 + \sin^4 \theta/2 + 2\cos^2 \theta/2 \sin^2 \theta/2}_{} =$

$= \underbrace{(\cos^2 \theta/2 + \sin^2 \theta/2)}_{1} = 1$

#) Найти в матр. форме опер. $\hat{S}_+$, $\hat{S}_-$

$S = 1/2$ $\quad \hat{S}_+^2; \hat{S}_-^2;$

$\hat{S}_+ = \hat{S}_x + i\hat{S}_y \quad - ?$

$\hat{S}_- = \hat{S}_x - i\hat{S}_y \quad - ?$

$\hat{S}_+^2 - ?$

$\hat{S}_-^2 - ?$

$\hat{S}_+ \cdot \psi_{S_z = \pm 1/2} \quad - ?$

$\hat{S}_- \psi_{S_z = \pm 1/2} - ?$

$$\hat{S}_z \Psi_\sigma = S_z^{\prime\prime 0} \Psi_\sigma$$

$$\hat{S}_z \hat{S}_\pm \Psi_\sigma = \left\{ \begin{array}{l} [\hat{S}_z, \hat{S}_x] \pm i [\hat{S}_z \hat{S}_y] \\ [\hat{S}_i, \hat{S}_k] = i \varepsilon_{ikl} \hat{S}_l \end{array} \right\} (2)$$

$$= i\hat{S}_y \pm \hat{S}_x = \pm(\hat{S}_x + i\hat{S}_y) = \underbrace{\pm \hat{S}_\pm} = [\hat{S}_z, \hat{S}_\pm] =$$

$$= (\pm \hat{S}_\pm + \hat{S}_\pm \hat{S}_z)\Psi_\sigma = \pm S_\pm \Psi_\sigma + \hat{S}_\pm \sigma \Psi_\sigma =$$

$$= \underbrace{(\sigma \pm 1)\hat{S}_\pm \Psi_\sigma = \hat{S}_z \hat{S}_\pm \Psi_\sigma}_{\sim \Psi_{\sigma \pm 1}} \qquad \hat{S}_z = \sigma \pm 1$$

$$\begin{cases} \hat{S}_+ \Psi_\sigma = c\Psi_{\sigma+1} \\ \hat{S}_- \Psi_\sigma = c\Psi_{\sigma-1} \end{cases}$$

#

$$(\hat{S}_x)_{\sigma,\sigma-1} = 1/2\sqrt{(s+\sigma)(s-\sigma+1)} = (\hat{S}_x)_{\sigma-1,\sigma}$$

$$(\hat{S}_y)_{\sigma,\sigma-1} = -i/2\sqrt{(s+\sigma)(s-\sigma+1)} = -(\hat{S}_y)_{\sigma-1,\sigma}$$

$(\hat{S}_z)_{\sigma\sigma} = \sigma\delta_{\sigma\sigma}$

$S = 1$

$\sigma = \underbrace{S, S-1, \ldots -S}_{2S+1}$

$1 \quad 0 \quad -1$

$$\hat{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

σ	i
1	1
0	2
-1	3

$$\hat{S}_x = \begin{pmatrix} 0 & \sqrt{2}/2 & 0 \\ \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & \sqrt{2}/2 & 0 \end{pmatrix}; \quad \hat{S}_y = \begin{pmatrix} 0 & -i\sqrt{2}/2 & 0 \\ i\sqrt{2}/2 & 0 & -i\sqrt{2}/2 \\ 0 & \frac{i\sqrt{2}}{2} & 0 \end{pmatrix}$$

$$\begin{cases} \hat{S}_z \Psi_\sigma = \sigma \Psi_\sigma \\ S\sigma | \hat{S}_z | \sigma S = \sigma < S\sigma | \sigma S > = \delta_{\sigma\sigma} \end{cases}$$

$$(\hat{S}_x)_{21} = (\hat{S}_x)_{\substack{1\,2 \\ \sigma \;\; \sigma-1 \\ \| \;\;\; \| \\ 1 \;\;\; 0}} = \frac{1}{2}\sqrt{\underbrace{\cancel{(S+\sigma)(S-\sigma+1)}}_{(1+1)(1-1+1)}} = \frac{\sqrt{2}}{2}$$

$$(\hat{S}_x)_{23} = (\hat{S}_x)_{32} = \frac{1}{2}\sqrt{(1+0)(1-0+1)} = \frac{\sqrt{2}}{2}$$

$\sigma = 0 \quad \sigma - 1 = -1$

$$(\hat{S}_y)_{12} = -(\hat{S}_y)_{21} = -i/2\sqrt{(1+1)(1-1+1)} = -i/2\sqrt{2}$$

$$(\hat{S}_y)_{23} = -(S_y)_{32} = -i/2\sqrt{(1+0)(1-0+1)} = -i/2\sqrt{2}$$

$$\hat{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \hat{S}_x = \frac{\sqrt{2}}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \hat{S}_y = \frac{\sqrt{2}}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\Psi_{S_z = 1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \Psi_{S_z = -1/2}(\vec{\tau}) \leftrightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\hat{S}_z>$$

$\langle \vec{r} \,|\, \hat{S}_z \rangle$

$\hat{S}_x \psi_{S_x} = S_x \psi_{S_x}$

$\psi_{S_x} \to a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$

$|a|^2 + |b|^2 = 1$ $\quad *$

$\frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = S_x \begin{pmatrix} a \\ b \end{pmatrix}$

$\frac{1}{2} \begin{pmatrix} b \\ a \end{pmatrix} = S_x \begin{pmatrix} a \\ b \end{pmatrix}$ $\quad \begin{cases} b/2 = S_x a \\ a/2 = S_x b \end{cases}$

$\begin{cases} b/2 = S_x 2 S_x b \\ a = 2 S_x b \end{cases} \Rightarrow \begin{cases} \cancel{b} = 4 S_x^2 \cancel{b} \\ a = 2 S_x b \end{cases} \Rightarrow S_x^2 = 1/4$

$\Downarrow$

$\hat{S}_x = \pm 1/2$

$S_x = \frac{1}{2}$ $\quad a = 2 \cdot 1/2\, b$

$\underline{a = b}$

$S_x = -\frac{1}{2}$ $\quad a = 2 \cdot -1/2\, b$

$\underline{a = -b}$

$b^2 + b^2 = 1$

$\Downarrow$

$2b^2 = 1$

$b^2 = \frac{1}{\sqrt{2}}$

$a = \pm b \Rightarrow b = \frac{1}{\sqrt{2}}$

из $(*)$

$\hat{S}_y \psi_{S_y} = S_y \psi_{S_y}$

$\psi_{S_y} \to c \begin{pmatrix} 1 \\ 0 \end{pmatrix} + d \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|d|^2 + |c|^2 = 1$ $\quad **$

$\frac{i}{2} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = S_y \begin{pmatrix} c \\ d \end{pmatrix}$

$\frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} c \\ d \end{pmatrix} = S_y \begin{pmatrix} c \\ d \end{pmatrix}$

$\frac{1}{2}\begin{pmatrix}-id\\ ic\end{pmatrix} = S_y \begin{pmatrix}c\\ d\end{pmatrix}$ $\begin{cases}-\frac{1}{2} id = S_y c\\ -\frac{1}{2} ic = S_y d\end{cases}$

$d = -2 S_y c/i$

$c = \frac{2}{i} S_y \left(-\frac{2}{i}\right) S_y \cancel{c} = +\frac{4}{1} S_y^2 \Rightarrow \frac{1}{4} = S_y^2$

$|c|^2 + |d|^2 = 1$ $S_y = \pm 1/2$

$|c|^2 + \left|4\left(\frac{1}{4}\right)c^2\right| = 1$ $2c^2 = 1$ $c = \frac{1}{\sqrt{2}}$

19.09.2012

$\langle \sigma S | \hat{S}_x | S \sigma - 1 \rangle$

$\langle \sigma S | \hat{S}_y | S \sigma - 1 \rangle$

$\hat{S}_+ \hat{S}_- = (\hat{S}_x + i\hat{S}_y)(\hat{S}_x - i\hat{S}_y) = \underbrace{\hat{S}_x^2 + \hat{S}_y^2}_{= \hat{S}^2 - \hat{S}_z^2} +$

$+ i[\hat{S}_y \hat{S}_x] = \hat{S}^2 - \hat{S}_z^2 + \hat{S}_z$

$i\varepsilon_{213} \hat{S}_z$, $\varepsilon_{213} = -1$

$\langle \hat{S}\sigma | \hat{S}_+ \hat{I} \hat{S}_- | \hat{S}\sigma \rangle = S(S+1) - \sigma^2 + \sigma + \underline{\sigma S - \sigma S} =$

$\hat{S}^2 |\hat{S}\sigma\rangle = S(S+1)|\hat{S}\sigma\rangle$

$\hat{S}_z |\hat{S}\sigma\rangle = \sigma|S\sigma\rangle$ $\langle S\sigma | S\sigma\rangle = 1$

$(S+\sigma)(S-\sigma+1)$

$I = \sum_{\sigma'} |S\sigma'\rangle\langle S\sigma'|$

$\langle S\sigma | \hat{I} | S\sigma\rangle = \sum_q \langle S\sigma | q\rangle\langle q | S\sigma\rangle$

$$\sum_{\sigma'} \langle S\sigma|\hat{S}_z|S\sigma'\rangle\langle S\sigma'|\hat{S}_-|S\sigma\rangle =$$
$$\sigma'=\sigma-1$$

$$= \langle S\sigma|\hat{S}_+|S\sigma-1\rangle\langle S\sigma-1|\hat{S}_-|S\sigma\rangle$$

$$= \left\{ \hat{S}_- = \hat{S}_+^{+} = \hat{S}_+^{*} \right\} = \langle S\sigma|\hat{S}_+|S\sigma-1\rangle \cdot$$

$$\cdot \langle S\sigma|\hat{S}_+|S\sigma-1\rangle^{*} = |\langle S\sigma|\hat{S}_+|S\sigma-1\rangle|^2 =$$

$$= |\langle S\sigma-1|\hat{S}|S\sigma\rangle|^2 = (S+\sigma)(S-\sigma+1)$$

$$(\hat{S}_+)_{\sigma,\sigma-1} = (\hat{S}_-)_{\sigma-1,\sigma} = \sqrt{(S+\sigma)(S-\sigma+1)}$$

$$\hat{S}_x = 1/2\,(\hat{S}_+ + \hat{S}_-)$$

$$\hat{S}_y = -\frac{i}{2}(\hat{S}_+ - \hat{S}_-)$$

$$(\hat{S}_x)_{\sigma,\sigma-1} = \frac{1}{2}\langle S\sigma|\hat{S}_+ + \hat{S}_-|S\sigma-1\rangle =$$

$$= \frac{1}{2}\langle S\sigma|\hat{S}_+|S\sigma-1\rangle + 1/2\underbrace{\langle S\sigma|\hat{S}_-|S\sigma-1\rangle}_{=0} =$$

$$= \frac{1}{2}\sqrt{(S+\sigma)(S-\sigma+1)} = (S_x)_{\sigma,\sigma-1}$$

$$(\hat{S}_y)_{\sigma,\sigma-1} = -i/2\,\langle\sigma|\hat{S}_+ - \hat{S}_-|\sigma-1\rangle =$$

$$= -i/2\,\langle\sigma|\hat{S}_+ - \hat{S}_-|\sigma-1\rangle = -i/2\sqrt{(S+\sigma)(S-\sigma+1)} =$$

$$= -(\hat{S}_y)_{\sigma-1,\sigma}$$

(1)

Модель Томаса-Ферми

$$r = a_0 \beta \cdot Z^{-1/3} x \qquad b = a_0\beta$$

$$\beta = \frac{1}{2}\left(\frac{3\pi}{4}\right)^{2/3}$$

плотность частиц

$$a_0 = \frac{\hbar^2}{me^2}$$

$$n(r) = \frac{32}{a_0^3} Z^2 \left(\frac{\Phi(x)}{x}\right)^{3/2}$$

$$\sqrt{x}\,\Phi'' = \Phi^{3/2}(x)$$

$$\begin{cases} \Phi(0) = 1 \\ \Phi(\infty) = 0 \end{cases}$$

$$\bar{f}(\vec{r}) = \frac{\int d^3r\, n(\vec{r}) f(\vec{r})}{\int d^3r\, n(\vec{r})}$$

Найти

$\langle r \rangle$; $\langle r^2 \rangle$ $\langle r^n \rangle_{n \geq 3}$ — ?

1) $$\langle r \rangle = \frac{\int d^3r\, n(\vec{r})\, r}{\int d^3r\, n(r)} = \frac{\int 4\pi r^3 n(r) dr}{\int 4\pi r^2 n(r) dr} =$$

$$= \frac{\int_0^\infty x^3 \left(\frac{\Phi(x)}{x}\right)^{3/2} dx}{\int_0^\infty x^2 \left(\frac{\Phi(x)}{x}\right)^{3/2} dx} \cdot a_0 \beta Z^{-1/3} \approx a_0 Z^{-1/3}$$

2) $$\langle r^2 \rangle = a_0^2 Z^{-2/3}$$

3) $$\langle r^n \rangle = \frac{\int d^3r\, n(r) r^n}{\int d^3r\, n(r)} = \frac{\int_0^\infty \cancel{4\pi} x^{n+2} \left(\frac{\Phi(x)}{x}\right)^{3/2} dx}{\int_0^\infty dx\, \cancel{4\pi} x^2 \left(\frac{\Phi(x)}{x}\right)^{3/2}} \cdot$$

$$\cdot\, a_0^n \beta_0^n Z^{-n/3}$$

из знамен. $\int_0^\infty \left(\frac{\Phi(x)}{x}\right)^{3/2} x^2 dx = \int_0^\infty \frac{\Phi'' x^2 dx}{x^{3/2}} = \int_0^\infty x\, \Phi''(x)\, dx =$

$= \int_0^\infty x\, d\Phi'(x) = \underline{x\,\Phi'(x)\Big|_0^\infty} - \int_0^\infty \Phi'(x) dx =$

$= 0 - \Phi(x)\Big|_0^\infty = 1$

из числит. $\int_0^\infty x^{n+2} \left(\frac{\Phi(x)}{x}\right)^{3/2} dx = F(\infty) - F(0)$

$F(x) \underset{x\to 0}{\sim} x^{n+2-3/2+1} > 0$

$x^{n+3/2} > 0$

$\underline{n > -3/2}$

$F(x) \underset{x\to\infty}{\sim} x^{n+2-3/2-9/2+1} < 0$

$\underline{n < 3}$

26.09.2012

Энергия взаимодействия с ядром

$z_1 = qe$

$z_2 = -e$

$E_{eN} = -e\varphi = -e\frac{ze}{r} = -\frac{ze^2}{r}$

$\langle f(\vec{r}) \rangle = \frac{\int d^3r\, n(\vec{r})\, f(\vec{r})}{\int d^3r\, n(\vec{r})}$

$\langle r^n \rangle \sim a^n e^{-n/3}$

$$\langle E_{en}\rangle = \langle -e\varphi\rangle = -e\left\langle \frac{Ze}{r}\right\rangle = \left\langle -\frac{Ze^2}{r}\right\rangle \sim$$

$$\sim \frac{Ze^2}{a_0} Z^{1/3} = \frac{Z^{4/3}e^2}{a_0} = \frac{Z^{4/3}e^4 m}{\hbar^2} = \boxed{\frac{me^4 Z^{4/3}}{\hbar^2}}$$

$$\left\langle \frac{1}{r}\right\rangle = a_0^{-1} Z^{1/3} \qquad a_0 = \frac{\hbar^2}{me^2}$$

Задача

найти среднее значение кинет энергии поля $\bar{e}$ и нейтр. в атоме и вычислить импульс

по т.е. Вириала

$$2\langle T\rangle = k\langle U\rangle$$

$$0 = \langle\Psi_n | [\hat{H}, \hat{F}] | \Psi_n\rangle = \langle \Psi_n | (\hat{H}\hat{F} - \hat{F}\hat{H}) | \Psi_n\rangle =$$

$$= \langle\Psi_n | \hat{H}\hat{F} | \Psi_n\rangle - \langle\Psi_n | \hat{F}\hat{H} | \Psi_n\rangle =$$

$$\hat{I} = \sum |\Psi_l\rangle\langle\Psi_l|$$

$$= \sum_l \langle\Psi_n | \underbrace{\hat{H} | \Psi_l\rangle}_{E_l \Psi_l} \langle \Psi_l | \hat{F} | \Psi_n\rangle - E_n \langle \Psi_n | \hat{F} | \Psi_n\rangle =$$

$$E_l \langle\Psi_n|\Psi_l\rangle$$

$$\delta_{nl}$$

$$= E_n\langle\Psi_n|\hat{F}|\Psi_n\rangle - E_n\langle\Psi_n|\hat{F}|\Psi_n\rangle = 0$$

$$\sum_n \langle\Psi_n | [\hat{H}, \hat{F}] | \Psi_n\rangle = \langle [\hat{H}, \hat{F}]\rangle = 0$$

$$\hat{F} = \vec{r}\hat{\vec{p}}$$

$$\langle [\hat{H}, \vec{r}\hat{\vec{p}}]\rangle = 0$$

$$[\hat{H}, \vec{r}\hat{\vec{p}}] = \underbrace{\left[\frac{\hat{p}^2}{2m}, \vec{r}\hat{\vec{p}}\right]}_{1} + [\hat{U}, \vec{r}\hat{\vec{p}}]$$

$$[\hat{\vec{p}}^2, \vec{r}\hat{\vec{p}}] = [\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2, x\hat{p}_x + y\hat{p}_y + zp_z] =$$

$$= p_x^2(x\hat{p}_x + y\hat{p}_y + z\hat{p}_z) - (-\text{''}-)\hat{p}_x^2 = [p_x^2\, x p_x]$$

аналогично и сум.

$$[p_x\, x] = p_x x \psi(x) - x p_x \psi(x) =$$

$$= -\hbar i \left[\left(\psi(x) + x\frac{\partial\psi}{\partial x}\right) - x\frac{\partial\psi}{\partial x}\right] = -i\hbar\psi(x)$$

$$\hat{p}_x^2 x\hat{p}_x - x\hat{p}_x\hat{p}_x^2 = \hat{p}_x \underbrace{\hat{p}_x x}_{xp_x - i\hbar}\hat{p}_x - x\hat{p}_x^3 =$$

$$= \underbrace{\hat{p}_x x}_{xp_x - i\hbar} p_x^2 - i\hbar p_x^2 - xp_x^3 = xp_x^3 - i\hbar p_x^2 - i\hbar p_x^2 - xp_x^3 =$$

$$= \boxed{-2i\hbar p_x^2}$$

$$\left\langle -\frac{ze^2}{r}\right\rangle - 2\left\langle\frac{p^2}{2m}\right\rangle = 0$$

$$\langle U\rangle \qquad\qquad 2\langle T\rangle$$

$$-\langle U\rangle - 2\langle T\rangle = 0$$

$$2\langle T\rangle = -1\langle U\rangle$$

$$\langle T\rangle = -\frac{1}{2} U$$

$$\langle T\rangle = +\frac{1}{2}\cdot ze^2 \left\langle\frac{1}{r}\right\rangle = \frac{ze^2}{2}\left\langle\frac{1}{r}\right\rangle$$

$$\left\langle\frac{1}{r}\right\rangle = a_0^{-1} z = \frac{1}{3}$$

$$\langle T\rangle = \frac{Ze^2}{2}a_0^{-1}Z^{1/3} = \frac{Z^{4/3}e^2}{2a_0}$$

$$\langle U\rangle = -2\langle T\rangle$$

$$\langle U\rangle = -\frac{1}{\cancel{2}m}\cancel{2}\langle\hat{p}^2\rangle \quad \frac{me^2Z^{4/3}}{2a_0}$$

$$\langle\hat{p}^2\rangle = \langle\hat{p}\rangle\langle\hat{p}\rangle = \frac{m^2e^4Z^{4/3}}{2\hbar^2}$$

$$\langle p\rangle \sim \frac{Z^{2/3}\hbar}{a_0} \sim \frac{e^2Z^{2/3}m}{\hbar}$$

$$a_0 = \frac{\hbar^2}{me^2}$$

$\langle$ Момент силы $\rangle$ – ?

$$\langle L\rangle \sim \langle r\rangle\langle p\rangle \sim Z^{2/3}\hbar$$

$\langle$ Энергия ионизации $\rangle$

$$I \sim Z\langle E_n\rangle \sim Z\langle e^2Z/r\rangle \sim Ze^2Z^{1/3}/a_0.$$

Энергия взаимодействия $\bar{e}$ с ватаме

$$\langle E_{вз}\rangle \sim \langle r\rangle = \frac{Z^{1/3}e^3}{a_0}$$

3.10.2012

$$\langle g(\vec{r}\vec{p})\rangle = \frac{\int d^3r\, d^3p\; f_{\vec{p}}(\vec{r})\, g(\vec{r},\vec{p})}{\int d^3r\, d^3p\; f_{\vec{p}}(\vec{r})}$$

$$n(\vec{r}) = \int dp\, f_{\vec{p}}(\vec{r})$$

$$dN = 2\,\frac{4\pi}{3}\frac{p_F^3(r)}{(2\pi\hbar)^3}\,dV$$

$$\vec{n}(r) = \frac{dN}{dV} = \frac{p_F^3}{3\pi^2\hbar^3} \qquad f_{\vec{p}}(\vec{r}) = -\eta(p_F - p)$$

$$p_F = (3\pi^2\hbar^3 n)^{1/3} \qquad \int n\, d^3r\, d^3p = Z$$

$$\eta(x) = \begin{cases} 1, & x>0 \\ 0, & x<0 \end{cases}$$

$$\langle p^n \rangle = \frac{\int d^3r \int_0^{p_F} dp\, \cancel{4\pi}\, p^{2+n}\, \eta}{\int d^3r \int_0^{p_F} dp\, \cancel{4\pi}\, p^2} = \frac{3\int d^3r\, p_F^{n+3}(r)}{(n+3)\int d^3r\, p_F^3(r)} =$$

$$= \left\{ \begin{aligned} r &= x a_0 Z^{-4} b^3 \\ n &= \frac{32\pi^2}{9}\frac{Z^2}{a_0^3}\left(\frac{\varphi}{x}\right)^{3/2} \end{aligned} \right\} = \frac{3}{n+3}\frac{\int_0^\infty x^2 dx\, (3\pi^2\hbar^3 n)^{\frac{n+3}{3}}}{\int_0^\infty x^2 dx\, (3\pi^2\hbar^3 n)} =$$

$$= \frac{3}{n+3}(3\pi^2\hbar^3)^{n/3}\frac{\int_0^\infty x^2 dx \left(\frac{32}{9\pi^2}\frac{Z^2}{a_0^3}\left(\frac{\varphi(x)}{x}\right)^{3/2}\right)^{\frac{n+3}{3}}}{\int_0^\infty x^2 dx\, \frac{32}{9\pi}\frac{Z^2}{a_0^3}\left(\frac{\varphi}{x}\right)^{3/2}} =$$

$$= \frac{3}{n+3}\left(3\cancel{\pi^2}\hbar^3\right)^{\frac{n}{3}}\left(\frac{32 Z^2}{9\pi^2 a_0^3}\right)^{n/3}\underbrace{\frac{\int_0^\infty x^2 dx\left(\frac{\varphi}{x}\right)^{n/2+3/2}}{\int_0^\infty x^2 dx\left(\frac{\varphi}{x}\right)^{3/2}}}_{=(*)} =$$

$\int_0^\infty x^2 dx\left(\frac{\varphi}{x}\right)^{3/2} \sim 1$

$$= \frac{3}{n+3}\left(\frac{32\hbar Z^{2/3}}{a_0}\right)^n \cdot (*)$$

$$(*) = \int_0^\infty x^2 dx\left(\frac{\varphi}{x}\right)^{3/2+n/2} = F(x)\Big|_0^\infty$$

$$F(x) \underset{x\to\infty}{\simeq} x^{2-4(n/2+3/2)+1} \quad \leq 0$$

$$-2n-3 \leq 0 \qquad n \geq -3/2$$

$$F(x) \underset{x\to 0}{\overset{\sim}{\Rightarrow}} x^{2-(n/2-3/2)+1} \quad >0 \quad \Rightarrow n<3$$

$$\langle p \rangle \sim \frac{\hbar Z^{2/3}}{a_0} \qquad \langle p^2 \rangle \sim \langle p \rangle^2$$

Спектральные термы уровни энергии в атоме

2

$E_{n,l}$ $(2l+1)(2s+1)$

$n = 1, 2$

$l = 0, 1, \ldots n-1$

$n, \; l = \begin{matrix} 0 & 1 & 2 & 3 \\ s & p & d & f \end{matrix}$

$(2p)^{k \leq 6}$ $k = \underset{3}{(2l+1)} \cdot \underset{2}{(2s+1)} = 6$ $1/2$

$n = 2$
$l = 1$

$(nl)^{k \leq (2l+1)(2s+1)}$

$\vec{L} = \sum_{i=1}^{2} \vec{L}_i$ $\vec{J} = \vec{L} + \vec{S}$

$\vec{S} = \sum_{i=1}^{2} \vec{S}_i$

L_1, L_2 $J = |L - S| \ldots |L + S|$

$L = |L_2 - L_1| \ldots |L_2 + L_1|$

$L_2 > L_1$ $2L_1 + 1$

$\boxed{\#1119}$ (a)

ns $n'p$

$n \neq n'$ $L_1 = 0$ $L_2 = 1$

$S_1 = 1/2$ $S_2 = 1/2$

$\vec{L} = \vec{L}_1 + \vec{L}_2$

$L\ |L_1 - l_2| \ldots L_1 + l_2$

$L = 1 \qquad \vec{S} = \vec{S}_1 + \vec{S}_2$

$S = \{0, 1\}$

$S\ |S_1 - S_2| \ldots |S_1 + S_2|$

$\vec{J} = \vec{L} + \vec{S}$

$J = |L - S| \ldots |L + S|$

сост. если $L = 1$ $S = 0$

$J = 1$

сост. $L = 1$ $S = 1$

$J = 0, 1, 2$

эли $L = 1$ $S = 0$; $L = 1$, $S = 1$ $\qquad L = 0\ 1\ 2\ 3$

$^1P_1 \qquad {}^3P_{0,1,2} \qquad$ S P D F

A $n = 1$ $\qquad L = 0 \ldots n - 1$

$L = 0 \qquad E_{n,L}\ (2L+1)(2S+1)$

$1S^2$

H $\quad 1S \quad L = 0 \quad S = 1/2$

$J = 1/2$

$^2S_{1/2}$

He $\quad (1S)^2 \quad L = 0; \quad S = 0, 1 \ldots; \; J = 0, 1 \Rightarrow {}^1S_0 \quad {}^3S_1$

$n=2$

Li $1s^2 2s$ $\uparrow_0$ $S = 1/2$ $L = 0$ $^2S_{1/2}$

$(1s)^2$

$2+6$ $(2s)^2 (2p)^6 = 2(\overset{n}{L}+1)^2 = 6$

$2+6$ $(3s)^2 (3p)^6 (3d)^{10} = (2\overset{n}{L}+1)\, 2 = 10$

$(4s)^2$

№ 11.19

б) $n \neq n'$

np $n'p$

$\vec{L} = 1;\ \vec{S} = 1/2$ $L_2 = 1$ $S_2 = 1/2$

$\vec{S} = \vec{S}_1 + \vec{S}_2$ $[0, 1]$

$\vec{L} = \vec{L}_1 + \vec{L}_2$ $[0, 1, 2]$

$|L - S| < \bar{J} \leq |S + L|$

$L = 0;$ $S = 0$

$\bar{J} = 0$ $^1S_0$

$L = 0$ $S = 1$

$\bar{J} = 1$ $^3S_1$

$L = 1$ $S = 0$

$\bar{J} = 1$ $^1P_1$

$L = 1$ $S = 1$

$\bar{J} = 2$ $^3P_{2,0,1}$

$L=1$, $S=2$ $\quad {}^5P_3$

$\bar{J}=3$

$L=0$, $S=2$

$\bar{J}=2$

$L=2$ $S=0$ $\quad J=0,1,2$

$S=1$ $\quad J=1,2,3$

Найдем состояние с наименьшей энергией (с наим. значением L)

${}^3D_{1,2,3}$

в - выполн дома

$\boxed{\text{Задача №11.20}}$

Найти возможные термы атомов

есть $2\bar{e}$ $(np)^2$
не могут находиться в один. кв сост с одинак. кв числами

$(np)^2$

$L_1=1$ $S_1=1/2$ $\quad S_{max}=1$; $S_{1z}=+1/2$ $S_2=+1/2$

$n_1=n_2$

$L_2=1$ $S_2=1/2$ $\quad L_{max}=2$ $\quad L_1=1$ $L_2=1$ $\quad n_2=n$

$\bar{e}$

1	2	L	S	
	а)	1	1/2	
	б)	0	1/2	
	в)	−1	1/2	а+б
	г)	1	−1/2	а+в
	д)	0	−1/2	а+г
	е)	−1	−1/2	а+д
				а+е

а)+а) - одинак кв. числа

ΣL_{iz}	ΣS_{iz}
1	1

$a+б$ ~~$(1,1)$~~ $б+в$ ~~$(-1,1)$~~ $в+г$ ~~$(0,0)$~~ $г+д$ $(1,-1)$

$a+в$ $(0,1)$ $б+г$ ~~$(1,0)$~~ $в+д$ ~~$(-1,0)$~~ $г+е$ $(0,1)$

$a+г$ ~~$(2,0)$~~ $б+д$ $(0,0)$ $в+е$ ~~$(-2,0)$~~ ~~$д+е$~~ $(-1,-1)$

$a+д$ ~~$(1,0)$~~ $б+е$ ~~$(-1,0)$~~

$a+е$ $(0,0)$

Получили 15 состояний $(L_z, S_z) \Rightarrow L, S, J$

$C_2^6 = 15$

1) $(\pm 2, 0)$ $(\pm 1, 0)$ $(0,0)$ $L=2$, $S=0$

$L_z \to L_{2L+1} \ldots L$

2) $S=1$ $(\pm 1; \pm 1)$ $(\pm 1, 0)$ $(0,0)$

$L=1$

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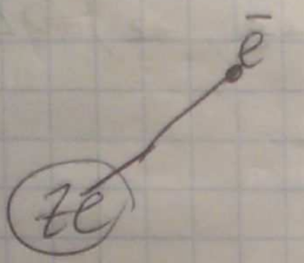
10.10.2012

Задача # 11-21

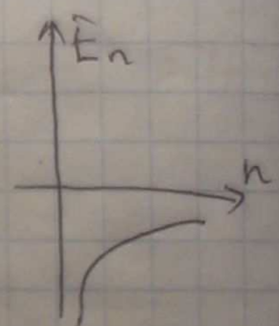
$R(r)$ – радиал. часть в/ф



$$R''(r) + \frac{2}{r}R'(r) + \frac{2\mu}{\hbar^2}\left(E - U_{eff}\right)R = 0$$

$$U_{eff} = -\frac{Ze^2}{r} + \hbar^2 l(l+1)/2\mu r^2$$

$$\mu = \frac{m_e M_я}{m_e + M_я} \approx m_e$$



$$E_n = -\frac{\mu Z^2 e^4}{2\hbar^2 n^2} \to -\frac{E_1}{n^2}$$

$E_1 =$

$n \geq l+1$

$0 \leq l \leq n-1$ $(2s+1)(2l+1)$

$(1s)^2$

$n=1$ $L=0$ $S_z = \pm\frac{1}{2}$ $(1s)^2$ $\cancel{(2s+1)(2l+1)}$

$n=2$ $L=0,1$ $(2s)^2(2p)^6$

$n=3$ $L=0,1,2$ $(3s)^2(3p)^6$

Совокупность электронов с один. кв. числами n и l – электр. оболочки

l

n 1s 2s 3s 4s 5s

2p 3p 4p 5p

3d 4d 5d

4f 5f

5g

← # 11.21

N $z=7$ N^+

Cl $z=17$ Cl^{7+}

$z=1$ H $1s$ $S=1/2$ $L=0$ $J=1/2$ $^2S_{1/2}$

$z=2$ He $1s^2$ $S=0$ $L=0$ $J=0$ $^1S_0$

$z=3$ Li $1s^2 2s$ $S=1/2$ $L=0$ $J=1/2$ $^2S_{1/2}$

$z=4$ Be $1s^2 2s^2$ $L=0$ $S=0$ $J=0$ $^1S_0$

$z=5$ B $1s^2 2s^2 2p^1$ $L=1$ $S=1/2$ $J=1/2$ $^2P_{1/2}$

$z=6$ C $1s^2 2s^2 2p^2$ $S=1$ $L=1$ $J=0$ $^3P_0$

$\uparrow\uparrow$
$1\ \ 0$

$z=7$ N $1s^2 2s^2 2p^3$ $S=3/2$ $L=0$ $J=3/2$ $^4S_{3/2}$ } Задача

$z=17$ Cl $\underline{1s^2 2s^2 2p^6} 3s^2 3p^5$ $S=1/2$ $L=1$ $J=3/2$ $^2P_{3/2}$

$z=8$ O $1s^2 2s^2 2p^4$ $S=1$ $L=1$ $J=2$ $^3P_2$

$\uparrow\downarrow\ \uparrow\ \uparrow$
$1\ \ 0\ -1$

$Z=9$	F	$1s^2 2s^2 2p^5$	$S=1/2$	$L=1$	$J=3/2$	$^2P_{3/2}$
$Z=10$	Ne	$\underbrace{1s^2 2s^2 2p^6}_{\star}$	$L=0$	$S=0$	$J=0$	$^1S_0$

N^+ $Z=7$ С " + " – убираем один $\bar{e}$

$1s^2 2s^2 2p^2$ $S=1$ $L=1$ $J=0$ $^3P_0$

Z^+ $1s^2 2s^2 2p^6 3s^2 3p^4$ $S=1$ $L=1$ $J=2$ $^3P_2$

$Z=11$	Na	$\star 3s^1$	$S=1/2$	$L=0$	$J=1/2$	$^2S_{1/2}$
$Z=12$	Mg	$3s^2$	$S=0$	$L=0$	$J=0$	$^1S_0$
$Z=13$	Al	$3s^2 3p^1$	$S=1/2$	$L=1$	$J=1/2$	$^2P_{1/2}$
$Z=14$	Si	$3s^2 3p^2$	$S=1$	$L=1$	$J=0$	$^3P_0$
$Z=15$	P	$3s^2 3p^3$	$S=3/2$	$L=0$	$J=3/2$	$^4S_{3/2}$
$Z=16$	S	$3s^2 3p^4$	$S=1$	$L=1$	$J=2$	$^3P_2$
$Z=18$	Ar	$3s^2 3p^6$	0	0	0	$^1S_0$
$Z=19$	K	$3s^2 3p^6 4s$	$S=1/2$	$L=0$	$J=1/2$	$^2S_{1/2}$
$Z=20$	Ca	$3s^2 3p^6 4s^2$	$S=0$	$L=0$	$J=0$	$^1S_0$
$Z=21$	Sc	$3s^2 3p^6 4s^2 3d$	$S=1/2$	$L=2$	$J=3/2$	$^2D_{3/2}$
$Z=22$	Ti	$3s^2 3p^6 4s^2 3d^2$	$S=1$	$L=3$	$J=2$	$^3F_2$
$Z=23$	V	$3s^2 3p^6 4s^2 3d^3$	$S=3/2$	$L=3$	$J=3/2$	$^4F_{3/2}$
$Z=24$	Cr	$\boxed{3s^2 3p^6 4s\, 3d^5}$	$S=5/2+1/2$	$L=0$	$J=5/2+1/2$	$^7S_3$

z=25	Mn	$3s^2 3p^6 4s^2 3d^5$	$S=5/2$	$L=0$	$J=5/2$		$^6S_{5/2}$
z=26	Fe	$3s^2 3p^6 4s^2 3d^6$	$S=2$	$L=2$	$J=4$		$^5D_4$
z=27	Co	$3s^2 3p^6 4s^2 3d^7$	$S=3/2$	$L=3$	$J=9/2$		$^4F_{9/2}$
z=28	Ni	$3s^2 3p^6 4s^2 3d^8$	$S=1$	$L=3$	$J=4$		$^3F_4$
z=29	Cu	$3s^2 3p^6 4s^1 3d^{10}$	$S=1/2$	$L=0$	$J=1/2$		$^2S_{1/2}$
z=30	Zn	$3s^2 3p^6 4s^2 3d^{10} \rightarrow *$	$S=0$	$L=0$	$J=0$	$\cancel{^1S_0}$	$^1S_0$
z=31	Ga	$3s^2 3p^6 4s^2 3d^{10} 4p$	$S=1/2$	$L=1$	$J=1/2$		$^2P_{1/2}$
z=32	Ge	$3s^2 3p^6 4s^2 3d^{10} 4p^2$	$S=1$	$L=1$	$J=0$		$^3P_0$
z=33	As	$* 4p^3$	$S=3/2$	$L=0$	$J=3/2$		$^4S_{3/2}$
z=34	Se	$4p^4$	$S=1$	$L=1$	$J=2$		$^3P_2$
z=35	Br	$4p^5$	$S=1/2$	$L=1$	$J=3/2$		$^2P_{3/2}$
z=36	Kr	$4p^6$	$S=0$	$L=0$	$J=0$	$\cancel{^1S_0}$	$^1S_0$

17.10.2012

Борновское приближение

3

Волновая функция $\psi = e^{ikz} + \frac{f(\theta)}{r} e^{ikr}$

Ур-ие Шред-и $U(\vec r)$

$$\Delta\psi + k^2\psi = \frac{2m}{\hbar^2} U(\vec r)\psi(\vec r)$$

$$\psi = \psi^{(0)} + \psi^{(1)}$$ ← пропорционально-расходящ. волна

$\psi^{(0)}$ — плоская волна

$$\Delta\psi^{(0)} + \psi^{(0)} k^2 = 0; \quad \psi^{(0)} = e^{\pm i\vec k\vec r}$$

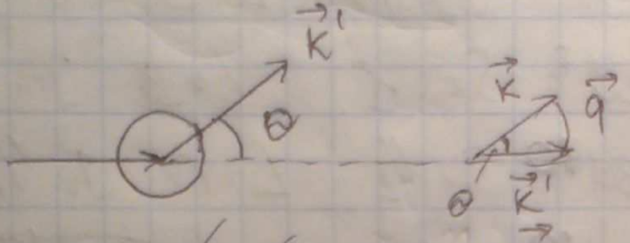
$$\boxed{f(\theta) = -\frac{m}{2\pi\hbar^2}\int U(\vec r)\, e^{i\vec q\vec r}\, d^3 r}$$

$f(q)$

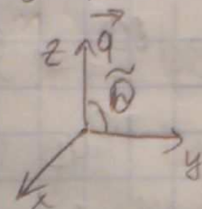
угол рассеяния

$d^3r = dr\, r^2 \sin\tilde\theta\, d\varphi\, d\tilde\theta$

$\tilde\theta$ — переменная интегрирования



$U(r, \cancel{\tilde\theta}, \cancel{\varphi})$



$\vec k = \vec k'$

$\vec q = \vec k - \vec k'$

$q = 2k \sin \theta/2$

$\frac{k^2\hbar^2}{2m} = E$

рассм. такие задачки

$$f(\vec q) = -\frac{m}{2\pi\hbar^2}\int_0^\infty r^2 dr \int_0^\pi d\tilde\theta \sin\tilde\theta \underbrace{\int_0^{2\pi} d\varphi}_{2\pi}\, \overbrace{e^{i\vec q\vec r}}^{=e^{iqr\cos\tilde\theta}} U(\vec r) =$$

$$= + \frac{m}{\hbar^2}\int_0^\infty r^2 dr\, U(\vec r)\int d\cos\tilde\theta \cdot e^{iqr\cos\tilde\theta}\cdot\frac{iqr}{iqr} =$$

$$= \frac{m}{\hbar^2}\int_0^\infty r^2 dr\, U(r)\, e^{iqr\cos\tilde\theta}\Big|_0^\pi = -\frac{mi}{\hbar^2 q}\int_0^\infty dr\, r\, U(r)\cdot$$

$$\cdot\frac{e^{-iqr} - e^{iqr}}{2\cancel{i}}\, 2\cancel{i} = \boxed{\frac{2m}{\hbar^2 q}\int_0^\infty dr\, r\, U(r)\sin qr = f(q)}$$

типа sin

$k = \frac{\sqrt{2mE}}{\hbar^2}$ $\quad q = 2k\sin\frac{\theta}{2}$

$\sigma = \int d\sigma = |f|^2 d\Omega$

полное сечение рассеяния

$\boxed{\#13.4}$ (a)

$U(r) = \alpha\,\delta(r-R)$

$$f(q) = -\frac{2m}{\hbar^2 q}\alpha\int_0^\infty dr\, r\,\delta(r-R)\sin qr = -\frac{2m\alpha}{\hbar^2 q}\cdot$$

$\cdot R\sin qR;$ $\left\{ \int dx\, f(x)\,\delta(x-a) = f(a) \right\}$

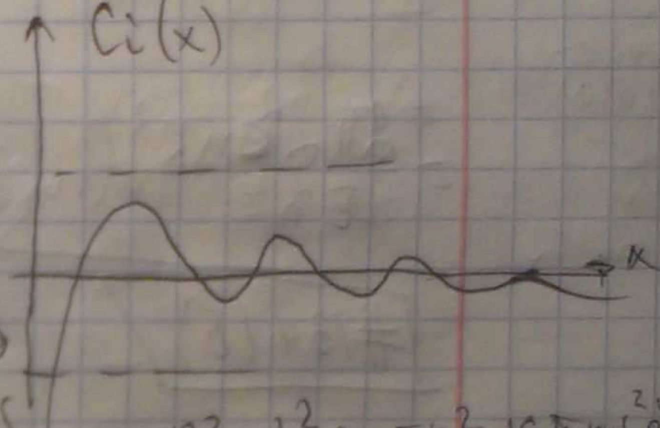
$$G(E) = \int -\frac{4m^2 \alpha^2 R^2}{\hbar^4 q^2} \sin qR \, d\varphi \cdot d\theta \sin\theta =$$

$$= \frac{8 \cancel{4} m^2 \alpha^2 R^2}{\hbar^4} \int \frac{\sin^2 2k \sin\frac{\theta}{2} R}{\cancel{4} k^2 \sin^2 \theta/2} \longrightarrow \frac{2\sin\frac{\theta}{2}}{\cancel{4}} \quad \cos\frac{\theta}{2}\frac{d\theta}{2}\frac{2kR}{2kR}$$

$$2kR = \sqrt{\frac{2m}{\hbar^2} E R^2}$$

$$\left\{ 2kR\sin\theta/2 = x \right\} = \frac{R^2 8 m^2 \alpha^2 \pi}{k^2 \hbar^4} \int_0^{2kR} dx \frac{\sin^2 x}{x} = \left\{ E = \frac{\hbar^2 k^2}{2m} \right\} =$$

$$= \frac{4\pi R^2 m \alpha^2}{\hbar^2 E} \int_0^{\sqrt{\frac{8mER^2}{\hbar^2}}} dx \frac{\sin^2 x}{x}$$



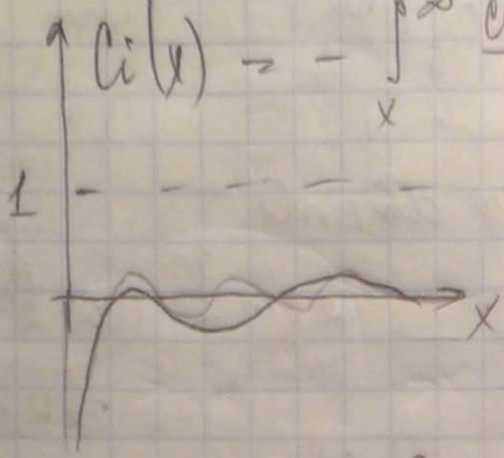
$$G(E) \xrightarrow{E\to\infty} \frac{8\pi \alpha^2 \pi R^2}{E\hbar^2} \int_0^{\sqrt{\frac{8mER^2}{\hbar^2}}} \frac{1-\cos 2x}{2x} dx^2 =$$

$$\frac{4\pi R^2 m\alpha^2}{2\hbar^2 E} \frac{8mER^2}{\hbar^2} = \frac{16\pi m^2 \alpha^2 R^4}{\hbar^4}$$

$$= \frac{2\pi\alpha^2 m R^2}{E\hbar^2} \left(\gamma + \ln x - Ci(x) \Big|_{x\to\infty} \right) =$$

$$= \frac{2\pi \alpha^2 m R^2}{E\hbar^2} \Big|_m \frac{8mEe^2}{\hbar^3}$$

$$Ci(x) = -\int_x^{\infty} \frac{\cos t}{t}\,dt = \gamma + \ln x - \int_0^{\infty} \frac{1-\cos t}{t}\,dt$$



$$\sigma(E) = \frac{2\pi \alpha^2 m R^2}{E\hbar^2}\left(\gamma + \ln x - \cancel{\cos}\, Ci(x)\right)\Big|_{x\to\infty} =$$

γ = 0,57 – пост. Эйлера

13.4(а)

$$= \frac{2\pi \alpha^2 m R^2}{E\hbar^2} \ln\left(8mER^2/\hbar^2\right)$$

#13.4(б)

$$U(r) = V_0 e^{-r/R}$$

$$\frac{d}{dq}\int \cos qr \cdot e^{-r/R}\,dr$$

$$f(q) = -\frac{2mV_0}{q\hbar^2}\int_0^{\infty} r \sin qr\, e^{-r/R}\,dr =$$

$\sin qr = \operatorname{Im} e^{iqr}$

$$= -\frac{2mV_0}{q\hbar^2}\int_0^{\infty} r \operatorname{Im} e^{iqr - r/R}\,dr = -\frac{2mV_0}{q\hbar^2} \cdot$$

$$\cdot \frac{d}{dq}\int_0^{\infty} \cos qr\, e^{-r/R} = -\frac{2mV_0}{q\hbar^2} \cdot$$

$$\cdot \frac{d}{dq} \operatorname{Re}\int_0^{\infty} e^{iqr - r/R}\,dr = +\frac{2mV_0}{q\hbar^2}\frac{d}{dq}\operatorname{Re}\frac{1}{\left(iq - \frac{1}{R}\right)} e^{iqr - r/R}$$

$$= +\frac{2mV_0}{q\hbar^2}\frac{d}{dq}\left(\frac{-iq + 1/R}{q^2 + \frac{1}{R^2}}\right) = \frac{2mV_0}{q\hbar^2}\frac{d}{dq}\left(\frac{1}{R\left(q^2 + 1/R^2\right)}\right) =$$

$$= -\frac{2mU_0}{q\hbar^2}\frac{1}{R}\frac{2q}{(q^2+1/R^2)^2} = \boxed{-\frac{4mU_0R^3}{\hbar^2((qR)^2+1)}}$$

$$U(r) = \frac{\alpha}{r}e^{-r/R}$$

$\boxed{\#134(b)}$ $U(r) = \frac{\alpha}{r}e^{-r/R}$

$$f(\theta) = -\frac{2m}{q\hbar^2}\int_0^\infty \frac{\alpha}{r}e^{-r/R}\cdot r\sin qr\,dr =$$

$$= -\frac{2m}{q\hbar^2}\alpha\int_0^\infty \sin qr\cdot e^{-r/R}dr = -\frac{2m}{q\hbar^2}\alpha\int_0^\infty e^{-r/R}\,\mathrm{Im}\,e^{iqr}dr =$$

$$= -\frac{2m}{q\hbar^2}\alpha\,\mathrm{Im}\int_0^\infty e^{iqr-r/R}dr = -\frac{2m}{q\hbar^2}\alpha\,\mathrm{Im}\frac{q}{(iq-1/R)}e^{r(iq-1/R)}\Big|_0^\infty$$

$$= \frac{2m}{q\hbar^2}\alpha\,\mathrm{Im}\left(\frac{-iq-1/R}{(-q^2-1/R^2)}\right) = -\frac{2m}{q\hbar^2}\alpha\,\mathrm{Im}\frac{iq+1/R}{q^2+1/R^2} =$$

$$= \frac{2m}{q\hbar^2}\alpha\frac{-q}{q^2+1/R^2} = \boxed{-\frac{2m\alpha}{\hbar^2(q^2+1/R^2)}}$$

#2 $U(r) = \frac{\alpha}{r^2}$

$$f(q) = -\frac{2m}{q\hbar^2}\int_0^\infty \frac{\alpha}{r^2}r\sin qr\,dr = -\frac{2m\alpha}{q\hbar^2}\int_0^\infty \frac{1}{r}\sin qr\,dr =$$

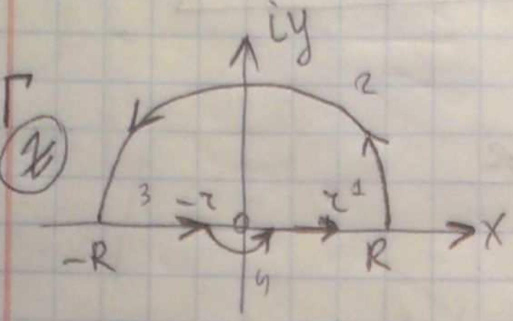
$$= -\frac{2m\alpha}{q\hbar^2}\,\mathrm{Im}\int_{-\infty}^\infty \frac{e^{iqr}}{r}dr \quad \pi$$

$$\oint_\Gamma f(z)dz = 2\pi i\sum_k \underset{z\to z_k}{\mathrm{res}}\, f(z)$$

Вычет для полюса $\sim n$

$$\underset{z\to z_k}{\overset{n}{\mathrm{res}}}\, f(z) = \frac{1}{(n-1)!}\lim_{z\to z_0}\frac{d^{n-1}}{dz^{n-1}}\left((z-z_0)^n f(z)\right)$$

$$f(z) = \frac{e^{iqz}}{z}$$



$\oplus$ ↺

$\ominus$ ↻

$$\oint_\Gamma \frac{e^{iqz}}{z}\,dz = \underbrace{\int \frac{e^{iqx}}{x}\,dx}_{\substack{z=x\\ ①}} + \underbrace{i\int_0^\pi d\varphi\, \cancel{\frac{Re^{i\varphi}}{Re^{i\varphi}}}\, e^{iqRe^{i\varphi}}}_{\substack{z=Re^{i\varphi}\\ dz=i\,d\varphi\, Re^{i\varphi}\\ ②}} +$$

$$+ \underbrace{\int_{-R}^{-r} dx\,\frac{e^{iqx}}{x}} +$$

24.10.2012

$$\oint \frac{e^{iqz}}{z}\,dz = 2\pi i \sum \operatorname{res}_{z\to 0} \frac{e^{iqz}}{z} = 1\left(z=x\right) + 2\left(\substack{z=Re^{i\varphi}\\ dz=i\,d\varphi\, Re^{i\varphi}}\right)$$

$$+\,3\left(z=x\right) + 4\left(\tfrac{1}{2}\oint f(z)\,dz\right) =$$

при $z\to\infty$ $\to 0$

$$= \cancel{\int_0^r \frac{e^{iqx}}{x}\,dx + \int_0^\pi Re^{i\varphi}\,d\varphi\cdot \underbrace{e^{iqR(\cos\varphi + i\sin\varphi)}}_{= e^{iqR\cdot\cos\varphi}\cdot e^{-iqR\sin\varphi}}} +$$

$$e^{-iqR\sin\varphi} \xrightarrow[z\to\infty]{} 0$$

$$+ \int_{-R}^{-r} \frac{e^{iqx}}{x}\,dx + \pi i \operatorname{res}_{z\to 0}\left(e^{iqz}/z\right) =$$

$$= \int_{-\infty}^{\infty} dx\, e^{iqx}/x = \pi i \lim \operatorname{res}_{z\to 0}\left(\frac{e^{iqz}}{z}\right) = \pi i \underbrace{\lim_{z\to 0} \cancel{z}\cdot \frac{e^{iqz}}{\cancel{z}}}_{1} =$$

$$= \pi i$$

$$\boxed{f(q) = -\frac{\pi i \alpha d}{q\hbar^2}}$$

#13.4 (д)

$$U(r)=\begin{cases}U_0, & r\le R\\ 0, & r>R\end{cases}$$

$$f(q)=-\frac{2m}{q\hbar^2}\int_0^\infty U(r)\,r\sin qr\,dr=\int_0^R U_0\,r\sin qr\,dr+$$

$$+\underbrace{\int_R^\infty 0\cdot r\sin qr\,dr}_{\to 0}=-\frac{2m}{q\hbar^2}U_0\frac{d}{dq}\int_0^R\cos qr\,dr=$$

$$=-\frac{2m}{q\hbar^2}U_0\frac{d}{dq}\left(\frac{q}{q}\sin qR\right)=-\frac{2m}{q\hbar^2}U_0\cdot$$

$$\cdot\cancel{\sin qR\,-\frac{1}{q^2}+R\cos qR\left(-\frac{1}{q^2}\sin qR\right)}$$

$$\left(-\frac{1}{q^2}\sin qR+\frac{1}{q}R\cos qR\right)=\boxed{-\frac{2m}{q\hbar^2}U_0\left(-\frac{1}{q^2}\sin qR+\frac{1}{q}R\cos qR\right)}$$

#13.4 (и)

$$U=U_0e^{-\frac{r^2}{R^2}}$$

$$f(q)=-\frac{m}{2\pi\hbar^2}\int U(\vec r)e^{i\vec q\vec r}d\vec r=-\frac{2mU_0}{2\pi\hbar^2}\iiint_{-\infty}^{+\infty}e^{-\frac{1}{R^2}(x^2+y^2+z^2)}\cdot$$

$$\cdot e^{-i(q_xx+q_yy+q_zz)}dx\,dy\,dz=-\frac{mU_0}{2\pi\hbar^2}\underbrace{\int_{-\infty}^{+\infty}e^{-\frac{x^2}{R^2}-iq_xx}dx}_{I(q_x)}\cdot I(q_y)\cdot$$

$$\cdot I(q_z)=$$

$$I(q_x)=\int_{-\infty}^{+\infty}e^{\left(-\frac{x^2}{R^2}-iq_xx\right)}dx=\int_{-\infty}^{+\infty}\exp\left\{-\frac{x^2}{R^2}+\cancel{iq_xx}+\frac{2iq_xx}{R}\cdot\frac{x}{R}-\frac{q_x^2R^2}{4}\right\}$$

$$\cdot e^{\left\{-\frac{q_x^2 R^2}{4}\right\}} dx = R \cdot e^{\left\{-\frac{q_x^2 R^2}{4}\right\}} \int_{-\infty}^{+\infty} e^{-\left(\frac{x}{R} + \frac{q_x R^2}{2}\right)^2} \cdot$$

$$\cdot d\underbrace{\left(\frac{x}{R} + \frac{q_x^2 R^2}{2}\right)}_{=t} = R \cdot e^{\left\{-\frac{q_x^2 R^2}{4}\right\}} \cdot \underbrace{\int_{-\infty}^{+\infty} e^{-t^2} dt}_{\sqrt{\pi}}$$

$$\int_{-\infty}^{+\infty} e^{-x^2} dx = \left(\int_{-\infty}^{+\infty} e^{-x^2} dx \int_{-\infty} e^{-y^2} dy\right)^{1/2} = \left(\iint_{-\infty}^{+\infty} e^{-(x^2+y^2)} dx dy\right)^{1/2}$$

$$= \left\{ \begin{matrix} x = \rho\cos\varphi \\ y = \rho\sin\varphi \end{matrix} \right\} \quad dx\,dy = \frac{d\rho\, d\varphi}{\left|\frac{\partial(\rho,\varphi)}{\partial(x,y)}\right|} = \rho\, d\rho\, d\varphi \Bigg\} *$$

$$\frac{\partial(\rho,\varphi)}{\partial(x,y)} = \left|\frac{\partial(x,y)}{\partial(\rho,\varphi)}\right|^{-1} = \left|\begin{matrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \varphi} \end{matrix}\right|^{-1} =$$

$$= \left|\begin{matrix} \cos\varphi & -\rho\sin\varphi \\ \sin\varphi & \rho\cos\varphi \end{matrix}\right|^{-1} = \rho^{-1} = \frac{1}{\rho}$$

$$* = \int_0^{\pi} d\varphi \int_0^{\infty} \rho\, d\rho\, e^{-\rho^2} = \left(2\pi \cdot \left(-\frac{1}{2}\right) \int_0^{\infty} d(-\rho^2) e^{-\rho^2}\right)^{1/2} =$$

$$= \sqrt{\pi\, \overline{e^{-\rho^2}}\Big|_0^{\infty}} = \sqrt{\pi}$$

$$I(q_x) = \sqrt{\pi} R \cdot e^{\left\{-\frac{q_x^2 R^2}{4}\right\}}$$

$$f(q) = -\frac{m U_0}{2\pi \hbar^2} \sqrt{\pi} \cdot R^3 e^{\frac{q^2 R^2}{4}} = \boxed{-\frac{m U_0 R}{2\sqrt{\pi}\hbar^2} R^3 e^{\frac{q^2 R^2}{4}}}$$

$\boxed{\#13.4(e)}$

$$U(r)=\begin{cases} U_0\left(\frac{r^2}{R^2}-1\right) & r\le R\\ 0 & r>R\end{cases}$$

$$f(q)=+\frac{2mU_0}{q\hbar^2}\frac{d}{dq}\int_0^\infty\left(\frac{r^2}{R^2}-1\right)\cos qr\,dr \ominus$$

$$\left\{\int U(r)\,r\sin qr\,dr = \frac{d}{dq}\int U(r)\cos qr\,dr\right\}$$

$$\ominus\ \frac{2mU_0}{q\hbar^2}\frac{d}{dq}\left(\int_0^R\frac{dr}{R^2}\left(\overset{=\frac{d^2}{dq^2}}{r^2\cos qr}\right)-\int_0^R\cos qr\,dr\right)=$$

$$=\frac{2mU_0}{q\hbar^2}\frac{d}{dq}\left\{-\frac{1}{R^2}\frac{d^2}{dq^2}\underbrace{\int_0^R\cos qr\,dr}_{-\frac{1}{q}\sin qR}-\frac{1}{q}\sin qR\right\}=$$

$$=\frac{4mU_0}{\hbar^2R^2q^5}\left(3\sin qR-3qR\cos qR-q^2R^2\sin qR\right)$$

$\boxed{\#13.4\,(з)}$ $U(r)=U_0e^{-ar}\left(1-e^{-br}\right)$

$$f(q)=-\frac{2m}{\hbar^2q}\int_0^\infty U(r)\,r\sin qr\,dr=$$

$$=-\frac{2mU_0}{\hbar^2q}\int_0^\infty r\sin qr\cdot e^{-ar}\left(1-e^{-br}\right)dr=$$

$$=-\frac{2mU_0}{\hbar^2q}\frac{d}{dq}\int_0^\infty\cos qr\cdot e^{-ar}\left(1-e^{-br}\right)dr=$$

$$=-\frac{2mU_0}{\hbar^2q}\frac{d}{dq}\int_0^\infty\underbrace{\cos qr}_{\mathrm{Re}\,(e^{iqr})}\cdot\left(e^{-ar}-e^{-r(a+b)}\right)dr=$$

$$= -\frac{2mU_0}{\hbar^2 q}\frac{d}{dq}\mathrm{Re}\int_0^\infty e^{iqr}\cdot\left(e^{-ar}-e^{-r(a+b)}\right)dr =$$

$$= -\frac{2mU_0}{\hbar^2 q}\frac{d}{dq}\mathrm{Re}\int_0^\infty \left(e^{(iq-a)r}-e^{-r(a+b+iq)}\right)dr =$$

$$= -\frac{2mU_0}{\hbar^2 q}\frac{d}{dq}\left[\mathrm{Re}\int_0^\infty e^{(iq-a)r}dr-\int_0^\infty e^{-r(a+b+iq)}dr\right] =$$

$$= -\frac{2mU_0}{\hbar^2 q}\frac{d}{dq}\mathrm{Re}\,(iq+a)$$

$$= -\frac{2mU_0}{\hbar^2 q}\frac{d}{dq}\mathrm{Re}\left(\frac{\cancel{iq+a}\;\cancel{q}+a}{q^2+a^2}+\frac{\cancel{R}-\cancel{iq}+(a+b)}{q^2+(a+b)^2}\right) =$$

$$= -\frac{2mU_0}{\hbar^2 q}\left(\frac{-a}{(q^2+a^2)^2}\cdot 2q+\frac{-(a+b)}{(q^2+(a+b)^2)^2}\cdot 2q\right) =$$

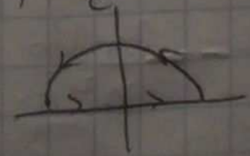
$$= -\frac{4mU_0}{\hbar^2}\left(\frac{a}{(q^2+a^2)^2}-\frac{(a+b)}{(q^2+(a+b)^2)^2}\right)$$

$\boxed{\#\ 13.4.\ (и)}$

$$U(r) = \frac{U_0}{1+r^2/R^2}$$

$$f(q) = -\frac{2mR^2U_0}{q\hbar^2}\frac{d}{dq}\int_0^\infty \cos qr/(R^2+r^2)\,dr =$$

$$= -\frac{\cancel{2}mU_0R^2}{q\hbar^2}\frac{d}{dq}\mathrm{Re}\int_{-\infty}^{\infty}\frac{e^{iqr}}{(R^2+r^2)}dr = \left\{\ \oint\frac{e^{iqz}}{z}dr = \right.$$



$$= \int_{-R}^{R} \frac{e^{-iqz}}{} + \int_{0}^{\pi}$$

$$= 2\pi i \cdot \operatorname*{res}_{z \to ia}$$

$$= \pi e^{-qa}$$

$$\Downarrow$$

$$= \frac{m R^2 U_0}{\hbar^2 q} \pi e^{-qa}$$

$$= \frac{R^2 m U_0}{\hbar^2 q} Re \left(2\pi i \lim_{z \to iR} (z - iR) \frac{e^{iqz}}{(z-iR)(z+iR)} \right)_{iq}$$

$$= \frac{m U_0 R^2}{q \hbar^2} Re \left(2\pi i \frac{e^{-qR}}{2iR} \right) = -\frac{\pi m U_0 R^2 e^{-qR}}{q \hbar^2}$$

1) ~~Вычислить работу~~
2) ~~приготовиться к контрольной~~

31.10.12 Теория фазы рассеяния

$r = 0$

13.7

$$\Delta_r R_n + k^2 R_n = 0$$

$$\Delta_r = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \dots \right) = \frac{1}{r} \frac{\partial^2}{\partial r^2} (r \dots)$$

$$\frac{\partial^2}{\partial r^2} r R_n + k^2 R_n r = 0$$

$$R_n \sim \frac{\sin(kr + \delta)}{r}$$

$$f(\theta) = \frac{1}{2ki} \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) (e^{2i\delta_l} - 1)$$

$$f_Б(\theta) = -\frac{2m}{q\hbar^2} \int_0^\infty dr\, r \sin qr\, U(r)$$

$$\delta_Б^l(k) - ?$$

$$= -\frac{2m}{\hbar^2}\int_0^\infty dr\, r^2 \frac{\sin qr}{qr} U(r) =$$

$$= -\frac{2m}{\hbar^2}\int_0^\infty dr\, r^2 U(r)\cdot \frac{\sin qr}{qr} = /^{A}$$

$$\boxed{\frac{\sin\sqrt{x^2+y^2-2xy\cos\theta}}{\sqrt{x^2+y^2-2xy\cos\theta}} = \sum_{l=0}^\infty (2l+1) P_l(\cos\theta)\, J_{l+\frac{1}{2}}(x)\cdot J_{l+\frac{1}{2}}(y)}$$

$A \; \left|\; q = 2k\sin\frac{\theta}{2} = \right.$

$$= 2k\sqrt{(1-\cos\theta)2} = \sqrt{2k^2-2k^2\cos\theta}$$

$$\frac{\sin qr}{qr} = \frac{\sin\left(\sqrt{k^2r^2+k^2r^2-2k^2r^2\cos\theta}\right)}{\sqrt{2k^2r^2-2k^2r^2\cos\theta}} =$$

$$y = x = kr \quad \frac{\pi}{2kr}\sum_{l=0}^\infty (2l+1)P_l(\cos\theta)\, J^2_{l+\frac{1}{2}}(kx) =$$

$$= -\frac{m\pi}{\hbar^2 k}\int_0^\infty dr\, r\, U(r)\left(\sum_{l=0}^\infty (2l+1)P_l(\cos\theta) J^2_{l+\frac{1}{2}}(kx)\right) =$$

$$= \sum_{l=0}^\infty (2l+1)P_l(\cos\theta)\left(-\frac{m\pi}{\hbar^2 k}\int_0^\infty dr\cdot r\, U\, J^2_{l+\frac{1}{2}}(kr)\right) =$$

$$= \frac{1}{2ik}\sum_{l=0}^\infty (2l+1)P_l(\cos\theta)\left(-\frac{2im\pi}{\hbar^2}\int_0^\infty dr\, r\, U\, J^2_{l+\frac{1}{2}}(kr)\right)$$

$$\boxed{-\frac{2im\pi}{\hbar^2}\int_0^\infty dr\, r\, U(r)\, J^2_{l+\frac{1}{2}}(kr) = 2i\delta_l^{B}}$$

#13.19

$U(r) = U_0 e^{-r/R}$

$kR \ll 1$

$\delta_l^{Б} = -\frac{\pi m}{\hbar^2} \int\limits_0^\infty dr\, r\, U(r)\, x^{2l+1}$

$J_\alpha(x) = \sum\limits_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+1+\alpha)} \left(\frac{x}{2}\right)^{2m+\alpha}$

$\alpha = l + \frac{1}{2}$, $x = kr$

$J_{l+\frac{1}{2}}(x) \approx \frac{1}{\Gamma(\frac{3}{2}+l)} \left(\frac{x}{2}\right)^{l+\frac{1}{2}} \simeq \frac{2^{-l-\frac{1}{2}}}{\Gamma(\frac{3}{2}+l)} x^{l+\frac{1}{2}}$

$\Gamma(z+1) = z\Gamma(z)$

$\delta_l^{Б} = -\frac{\pi m}{\hbar^2} \int\limits_0^\infty dr\, r\, U(r) (kr)^{2l+1} \cdot \frac{1}{\Gamma^2(\frac{3}{2}+l)\, 2^{2l+1}} =$

$= -\frac{\pi m}{\hbar^2} \left(\frac{k}{2}\right)^{2l+1} \int\limits_0^\infty dr\, U(r)\, r^{2l+2}$

Быстрые частицы

$x \to \infty$

Выражение асимптотики Бесселя

$J_\alpha(x) = \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right)$

#13 20

$$r \cdot U(r) \xrightarrow{r\to 0} 0$$

$$\delta_l^{Б} = -\frac{\pi m}{\hbar^2}\int_0^\infty dr\, r\, U(r)\, \frac{2}{\pi k r}\cos^2\left(kr - \frac{(l+\frac{1}{2})\pi}{2} - \frac{\pi}{4}\right)$$

$$\delta_l^{Б} \sim k^{2l+1} = -\frac{m\pi}{\hbar^2}\int_0^\infty dr\, r\, U(r)\frac{1}{\pi k r}\left(1 - \cos(2kr - l\pi)\right) =$$

$$= -\frac{m\pi}{\hbar^2}\int_0^\infty dr\, U(r)\frac{1}{\pi k} = -\frac{m}{\hbar^2 k}\int_0^\infty dr\, U(r)$$

$$U(r) \sim \frac{1}{r^\alpha} \qquad \frac{1}{r^{1+\alpha}}$$

Для S волн

$l = 0$

$U(r) = U_0 e^{-r/R}$

$$\delta_l^{Б} = -\frac{m\pi}{\hbar^2}\int_0^\infty dr\, r\, U_0 e^{-\frac{r}{R}}\, J^2_{1/2}(kr) = \Big|^{B}$$

$$J_{\frac{1}{2}}(x) = \sum_{m=0}^\infty \frac{(-1)^m}{m!\,\Gamma(m+\frac{3}{2})}\left(\frac{x}{2}\right)^{2m+\frac{1}{2}}$$

$$\Gamma\left(m+\frac{3}{2}\right) = \left(m+\frac{1}{2}\right)\Gamma\left(m+\frac{1}{2}\right) = \left(m+\frac{1}{2}\right)\left(m-1+\frac{1}{2}\right)\cdot$$

$$\cdots\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\right) = \left(\frac{2m+1}{2}\right)\left(\frac{2m-1}{2}\right)\cdots\left(\frac{1}{2}\right)\sqrt{\pi} =$$

$$= \frac{(2m+1)!!}{2^{m+1}}\sqrt{\pi}$$

$$J_{1/2}(x) = \sum_{m=0}^\infty \frac{(-1)^m\, 2^{m+1}\, x^{2m+\frac{1}{2}}}{m!\,(2m+1)!!\,\sqrt{\pi}\, 2^{2m+\frac{1}{2}}} =$$

$$= \sum_{m=0}^{\infty} \frac{(-1)^m \sqrt{2}\, x^{2m+\frac{1}{2}}}{\underbrace{(m!\,2^m)}_{(2m)!!}(2m+1)!!\,\sqrt{\pi}} = \sqrt{\frac{2}{\pi x}} \cdot \underbrace{\sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)!} x^{2m+1}}_{\sin x}$$

$$B = -\frac{2}{\pi}\frac{m\pi}{\hbar^2 k}\int_0^{\infty} dz\, z U_0 e^{-\frac{z}{R}} \frac{1}{z}\sin^2(kz) =$$

$$= -\frac{2m}{\hbar^2 k} U_0 \int_0^{\infty} dz\, e^{-\frac{z}{R}}\left(1-\cos(2kz)\right) =$$

$$\cancel{= + \frac{m}{\hbar^2 k} U_0 R + \frac{m}{\hbar^2 k} U_0 \operatorname{Re} \frac{1}{-\frac{1}{R}+2ik}}$$

$$= \frac{m}{\hbar^2 k} U_0 R \int_0^{\infty} e^{-\frac{z}{R}}\, d\left(-\frac{z}{R}\right) +$$

$$+ \frac{m}{\hbar^2 k} U_0 \operatorname{Re} \int_0^{\infty} dz\, e^{z\left(-\frac{1}{R}+2ik\right)} =$$

$$= -\frac{m}{\hbar^2 k} U_0 R + \frac{m}{\hbar^2 k} U_0 \operatorname{Re} \frac{1}{-\frac{1}{R}+2ik} \cdot$$

$$\cdot \int_0^{\infty} d\left(z\left(-\frac{1}{R}+2ik\right)\right) e^{z\left(-\frac{z}{R}+2ik\right)} =$$

$$= -\frac{m}{\hbar^2 k} U_0 R - \frac{m}{\hbar^2 k} U_0 \left(-\frac{1}{R}\right)\frac{1}{\frac{1}{R^2}+4k^2} =$$

$$= -\frac{m}{\hbar^2 k} U_0 \left(-\frac{1}{R}\right)\frac{R^2}{1+4k^2R^2} = -\frac{m U_0}{\hbar^2 k} R\left(1-\frac{1}{1+4k^2R^2}\right)$$

$$= -\frac{m U_0 R}{\hbar^2 k}\left(\frac{4k^2R^2}{1+4k^2R^2}\right) = \delta_0^{B}$$

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta)\left(e^{2i\delta_l} - 1\right) =$$

$$= \frac{1}{2ik} \cdot 1 \cdot 1 \cdot 2i\delta_0^{\text{Б}} = \frac{\delta_0^{\text{Б}}}{k}$$

$$\sigma = \int d\Omega \, |f|^2 = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \, \frac{|\delta_0^{\text{Б}}|^2}{k^2} =$$

$$= -\frac{16\pi m U_0 R_l^3}{\hbar^2 (1 + 4k^2 R_l^2)} \cdot 4\pi \frac{k^2}{k^2} -$$

15.17   07.11.12

$N = 2$, $s_{1,2} = \frac{1}{2}$

$\varphi_{s_1 s_2}$ $\vec{S} = \vec{S}_1 + \vec{S}_2$

$$|s_1 - s_2| \le S \le s_1 + s_2$$

$$0 \le S \le 1$$

1 ч. φ_{s_z} ; $\varphi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\varphi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

2 ч. χ_{s_z} $\chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2$, $\chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2$

$S=0$; $S=1$, $S_z=-1, 0, 1$

① $S_{z_1}=\frac{1}{2}$ ② $S_{z_2}=-\frac{1}{2}$

$S_z=-1$, $S_{z_1}=-\frac{1}{2}=S_{z_2}$

$S_z=1$, $S_{z_1}=\frac{1}{2}=S_{z_2}$

① $S_{z_1}=1-\frac{1}{2}$ ② $S_{z_2}=\frac{1}{2}$

$S_z=0$, $S_{z_1}=-\frac{1}{2}$ $S_{z_2}=\frac{1}{2}$

$S_{z_1}=\frac{1}{2}$ $S_{z_2}=-\frac{1}{2}$

$$\boxed{\psi_1=\varphi_{1/2}\,\chi_{1/2}=\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2}$$

ψ_{10}

ψ_{00}

$S_z=0$ $\psi_{00}=C_1\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2+C_2\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2$

$$\boxed{|C_1|^2+|C_2|^2=1}$$

$$\vec{S}^2\psi_{00}=S(S+1)\psi_{00}=\{e\,S=0\}=0=$$

$$=(\hat{S}_x^2+\hat{S}_y^2+\hat{S}_z^2)\psi_{00};$$

$$\left(\varphi_{00}, \vec{S}^2 \varphi_{00}\right) = \left(\varphi_{00}, \left(\hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2\right)\varphi_{00}\right) =$$

$$= \left(\varphi_{00}, \hat{S}_x^2 \varphi_{00}\right) + \ldots = \left(\hat{S}_x \varphi_{00}, \hat{S}_x \varphi_{00}\right) + \qquad \hat{S}^{+} = \hat{S}$$

$$\boxed{\left(\varphi, A\varphi\right) = \left(B\varphi, \varphi\right)}$$

$$= |\hat{S}_x|^2 |\varphi_{00}|^2 + |\hat{S}_y \varphi_{00}|^2 + |\hat{S}_z \varphi_{00}|^2 = 0$$

$$\hat{S}_x \varphi_{00} = 0, \quad \hat{S}_x = \underbrace{\hat{S}_{x_1}}_{=\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}_1} + \underbrace{\hat{S}_{x_2}}_{=\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}_2}$$

$$\frac{1}{2}\left(\begin{pmatrix}0&1\\1&0\end{pmatrix}_1 + \begin{pmatrix}0&1\\1&0\end{pmatrix}_2\right)\cdot\left(c_1\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + c_2\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right) = 0$$

$$\frac{1}{2}\Big[c_1\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + c_2\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 + c_1\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 +$$

$$+ c_2\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2\Big) = 0$$

$$\frac{1}{2}(c_1 + c_2)\Big(\underbrace{\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2}_{=\varphi_{1,-1}} + \underbrace{\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2}_{=\varphi_{1,1}}\Big) = 0$$

$$\begin{cases} c_1 + c_2 = 0 \\ |c_1|^2 + |c_2|^2 = 1 \end{cases} \qquad c_1 = \frac{1}{\sqrt{2}} = -c_2$$

$$\varphi_{00} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right)$$

φ_{10} — ?

$$\varphi_{10} = \left(D_1\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + D_2\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right)$$

$$\vec{S}^2\varphi_{10} \neq 0 = S(S+1)\varphi_{10}$$

$$|D_1|^2 + |D_2|^2 = 1$$

$$0 = (\varphi_{10}, \varphi_{00}) = \frac{1}{\sqrt{2}}\left(\left(D_1\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + D_2\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right),\left(\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right)\right) =$$

$$= \frac{1}{\sqrt{2}}(D_1 - D_2) = 0$$

$$\begin{cases} D_1^* - D_2 = 0 \\ |D_1|^2 + |D_2|^2 = 1 \end{cases} \qquad D_1 = D_2 = \frac{1}{\sqrt{2}}$$

$$\varphi_{10} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right)$$

$$\varphi_{11} = \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2$$

Найти дома по второму способу дома

$$\hat{S}_{\pm} = \hat{S}_{x_1} + \hat{S}_{x_2} \pm i(\hat{S}_{y_1} + \hat{S}_{y_2}) = \begin{pmatrix} 0 & \pm 1 \\ 0 & 0 \end{pmatrix}_1 + \begin{pmatrix} 0 & \pm 1 \\ 0 & 0 \end{pmatrix}_2$$

$$\hat{S}_{x_1} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{S}_{y_1} = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\cancel{S_+ \psi_{00} = \{\psi_{01} = \nexists\}}$$

$$S_+ \psi_{00} = \{ \underbrace{\psi_{01}}_{\substack{S=1, \\ \text{но} \\ S=0}} = \nexists \} = 0 = \left[\begin{pmatrix} 0 & \pm 1 \\ 0 & 0 \end{pmatrix}_1 + \begin{pmatrix} 0 & \pm 1 \\ 0 & 0 \end{pmatrix}_2 \right] \cdot$$

$$\cdot \left(c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right) =$$

$$= \left(c_1 \begin{pmatrix} 0 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right) = 0$$

$$\begin{cases} c_1 + c_2 = 0 \\ |c_1|^2 + |c_2|^2 = 1 \end{cases}$$

14.11.12

5.18

$$\hat{\vec{\sigma}}_1 \quad \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 \qquad \hat{\vec{S}}^2 \psi_{S S_z} = S(S+1)\psi_{S S_z}$$

$$\hat{\vec{S}} = \hat{\vec{S}}_1 + \hat{\vec{S}}_2, \quad \hat{\vec{S}}^2 = \hat{\vec{S}}_1^2 + \hat{\vec{S}}_2^2 + 2\hat{\vec{S}}_1\hat{\vec{S}}_2$$

$$= \hat{\vec{S}}_1^2 + \hat{\vec{S}}_2^2 + \frac{1}{2}\hat{\vec{\sigma}}_1\hat{\vec{\sigma}}_2$$

$$\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 \psi_{S S_z} = 2(\hat{\vec{S}}^2 - \hat{\vec{S}}_1^2 - \hat{\vec{S}}_2^2)\psi_{S S_z} =$$

$= 2S(S+1)\varphi_{SS_z} - 2\sum \alpha_{ij}\varphi_j \hat{\vec{S}}_1^2 \varphi_i - 2\sum \alpha_{ij}\varphi_i \hat{\vec{S}}_2^2 \chi_j =$

$$\boxed{\begin{aligned}\varphi_{SS_z} &= \sum \alpha_{ij}\varphi_i \chi_j \\ \varphi_{11} &= \varphi_1 \chi_1 = \begin{pmatrix}1\\0\end{pmatrix}_1 \begin{pmatrix}1\\0\end{pmatrix}_1\end{aligned}} \qquad \boxed{\hat{\vec{S}}_1^2 \varphi_1 = s_1(s_1+1)\varphi_1 \} = \frac{3}{4}\varphi_1}$$

$= 2S(S+1)\varphi_{SS_z} - \frac{3}{2}\varphi_{SS_z} - \frac{3}{2}\varphi_{SS_z} =$

$= 2S(S+1)\varphi_{SS_z} - 3\varphi_{SS_z} = [2S(S+1) - 3]\varphi_{SS_z} =$

$= \begin{cases} -3\varphi_{SS_z}, & S=0 \\ \varphi_{SS_z}, & S=1 \end{cases}$

5.20

$(\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)^2 - ?$

$(\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)^2 \varphi_{SS_z} = \left\{ \begin{aligned} &(\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 + 3)(\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 - 1)\varphi_{SS_z} = 0; \\ &(\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)^2 + 2\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 - 3 = 0 \end{aligned} \right\} =$

$= (3 - 2\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)\varphi_{SS_z} = \begin{cases} 9, & S=0 \\ 1, & S=1 \end{cases}$

$\langle \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 \rangle = \langle \varphi_{SS_z} | \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 | \varphi_{SS_z} \rangle =$

$= \langle \varphi_{SS_z} | \varphi_{SS_z} \rangle \begin{cases} -3, & S=0 \\ 1, & S=1 \end{cases}$

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$$\langle (\vec{\sigma}_1 \vec{\sigma}_2)^2 \rangle$$

$s = 1$

$$|\psi_{ss_z}\rangle = c_1 |\psi_{11}\rangle + c_2 |\psi_{10}\rangle + c_3 |\psi_{1-1}\rangle$$

$$\langle \psi_{ss_z}| = c_1^* \langle \psi_{11}| + c_2^* \langle \psi_{10}| + c_3^* \langle \psi_{1-1}|$$

$\langle \psi_{1i} | \psi_{1j} \rangle = \delta_{ij}$ ← нормировка

$$\langle \psi_{ss_z} | \psi_{ss_z} \rangle = |c_1|^2 + |c_2|^2 + |c_3|^2 = 1$$

Итого: $\langle (\sigma_1 \sigma_2)^2 \rangle = \begin{cases} 9 & s=0 \\ 1 & s=1 \end{cases}$

#5.22
исп. ответ из #5.18

$\{\varphi_n\}$ $\quad (\varphi_n, \varphi_m) = \delta_{nm}$

$$\hat{P}_n \varphi_m = \delta_{nm} \varphi_m = \begin{cases} 0 & n \neq m \\ \varphi_n & m = n \end{cases}$$

1) $\hat{P}_n \psi = \left\{ \psi = \sum_{i=1}^{N} c_i \varphi_i \right\} = \sum_{i=1}^{N} c_i \hat{P}_n \varphi_i = c_n \varphi_n$

2) $\hat{P}_n^+ = \hat{P}_n$ $\quad (\varphi_k, \hat{P}_n \varphi_m) = \ldots$

$(P_n \varphi_k, \varphi_m) = \ldots$

$(\psi, \hat{B}\psi) = (\hat{A}\psi, \psi)$
$\hat{B}$ сопряжен с $\hat{A}$

3) $\hat{P}_n \hat{P}_m = \delta_{nm} \hat{P}_m$

4) $\sum_n \hat{P}_n = \hat{1} \Rightarrow \sum_n \hat{P}_n \psi_k = \psi_k$

5) $\hat{f} = \sum_n f_n \hat{P}_n \Rightarrow \left(\sum_m f_m \hat{P}_m\right) \psi_k = f_k \psi_k = \hat{f} \psi_k$

6) $F(\hat{f}) \psi_n = F(f_n) \psi_n \Rightarrow$ через ~~розклад~~ розкл. в ряд Тейлора

$F(\hat{f}) = \sum_n F(f_n) \hat{P}_n$

7) $\hat{P}_n = |\psi_n\rangle\langle\psi_n| \Rightarrow \hat{P}_n |\psi_m\rangle =$

$= |\psi_n\rangle\langle\psi_n|\psi_m\rangle =$

$= \delta_{mn} |\psi_n\rangle$

$\hat{1} = \sum_n |\psi_n\rangle\langle\psi_n|$

5.22

$\hat{P}$ – проекційний оператор

$P_{S=0} - ?, \quad P_{S=1} - ?$

$\hat{P}_0 = \alpha \hat{\vec{S}} - \beta = \gamma \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 - \delta$

$$\begin{cases} 1.\ \hat{P}_0 \psi_{00} = \psi_{00} \\ 2.\ \hat{P}_0 \psi_{1 S_z} = 0 \\ 3.\ \hat{P}_1 \psi_{1 S_z} = \psi_{1 S_z} \\ 4.\ \hat{P}_1 \psi_{00} = 0 \end{cases}$$

1. $\hat{P}_0 \psi_{00} = (\alpha \hat{\vec{S}} - \beta) \psi_{00} = (\gamma \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 - \delta) \psi_{00} =$

$= \gamma(-3) - \delta = 1$

2. $\gamma \cdot 1 - \delta = 0$

3. из (1) и (2)

4.
$$\begin{cases} -3\gamma - \delta = 1 \\ \gamma - \delta = 0 \end{cases} \quad \begin{cases} -4\gamma = 1 \\ \gamma - \delta = 0 \end{cases} \quad \begin{cases} \gamma = -\frac{1}{4} \\ \gamma = \delta \end{cases}$$

$$\boxed{\hat{P}_0 = -\frac{1}{4}\hat{\vec{\sigma}}_1\hat{\vec{\sigma}}_2 + \frac{1}{4}}$$

$P_1 \psi \quad \hat{P}_1 = \gamma_1 \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 - \delta_1$

1) $\hat{P}_1 \psi_{00} = (\gamma_1 \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 - \delta_1)\psi_{00} = 0$

$-3\gamma_1 - \delta_1 = 0$

2) $\hat{P}_1 \psi_{1 s_z} = (\gamma_1 \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 - \delta_1)\psi_{1 s_z} = 1$

$\gamma_1 - \delta_1 = 1$

$$\begin{cases} -3\gamma_1 - \delta_1 = 0 \\ +\gamma_1 - \delta_1 = 1 \end{cases} \quad \begin{cases} \delta_1 = -3\gamma_1 \\ 4\gamma_1 = 1 \end{cases} \quad \begin{cases} -\delta_1 = -\frac{3}{4} \\ \gamma_1 = \frac{1}{4} \end{cases}$$

$$\boxed{\hat{P}_1 = \frac{1}{4}\hat{\vec{\sigma}}_1\hat{\vec{\sigma}}_2 + \frac{3}{4}}$$

$$\hat{P}_{00} = \hat{P}_0 = \frac{(1 - \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)}{4}$$

$\hat{P}_{11} \psi_{11} = \psi_{11}$

$\hat{P}_{11} \psi_{1-1} = 0$

$\hat{P}_{11} \psi_{10} = 0$

$\hat{P}_{11} \psi_{00} = 0$

$\hat{P}_{11} \psi = \alpha \hat{S}_z (\hat{S}_z + 1)\psi_n$

$\hat{P}_{11} \psi_{11} = 2\alpha \psi_{11} - \psi_{11}$

$$\hat{P}_{11} + \hat{P}_{10} + \hat{P}_{1-1} + \hat{P}_{00} = 1$$

$$\hat{P}_{1-1} \Psi_{ss_z} = \beta \hat{S}_z (S_z - 1) \Psi_{ss_z}$$

$$\hat{P}_{1-1} \Psi_{1-1} = 2\beta \Psi_{1-1} = \Psi_1$$

$$2\beta = 1 \qquad \boxed{\beta = \frac{1}{2}}$$

$$P_{11} = \frac{1}{2} \hat{S}_z (\hat{S}_z + 1) \quad P_{00}$$

$$\hat{P}_{1-1} = \frac{1}{2} \hat{S}_z (\hat{S}_z - 1)$$

$$\left[\frac{1}{2} \hat{S}_z (\hat{S}_z + 1) + \frac{1}{2} \hat{S}_z (\hat{S}_z - 1) + \right.$$

$$\left. + \left(\frac{1 - \hat{\sigma}_1 \hat{\sigma}_2}{4} \right) \right] - 1 = P_{10}$$

$$\hat{S}_z^2 = (\hat{S}_{1z} + \hat{S}_{2z})^2 = \hat{S}_{1z}^2 + 2 \hat{S}_{1z} \hat{S}_{2z} + \hat{S}_{2z}^2$$

$$\boxed{\hat{P}_{10} = \frac{1 - 2\hat{\sigma}_z \hat{\sigma}_{2z} + \vec{\hat{\sigma}}_1 \vec{\hat{\sigma}}_2}{4}}$$

~~# 5.28~~ ???

28.11.12

$$\hat{A} = a_0 \hat{I} + \underbrace{a_1 \hat{\sigma}_x + a_2 \hat{\sigma}_y + a_3 \hat{\sigma}_z}_{\vec{a}\hat{\vec{\sigma}}} = a_0 \hat{I} + \vec{a}\hat{\vec{\sigma}} =$$

$$Sp\, \hat{\sigma}_k = 0$$

$$= \hat{A}$$

$$\hat{\sigma}_i \hat{\sigma}_k = \delta_{ik} + i \varepsilon_{ikl} \hat{\sigma}_l$$

1) $Sp(a_0 \hat{I} + \vec{a}\hat{\vec{\sigma}})\hat{\sigma}_k = Sp(a_0 \hat{\sigma}_k + a_i \hat{\sigma}_i \hat{\sigma}_k) =$

$$= a_0 \underset{0}{\underset{\parallel}{Sp\, \hat{\sigma}_k}} + Sp(a_1 \hat{\sigma}_1 \hat{\sigma}_k + a_2 \hat{\sigma}_2 \hat{\sigma}_k + a_3 \hat{\sigma}_3 \hat{\sigma}_k =$$

$$= a_i \delta_{ik}\, Sp\, \delta_{ik} = 2 a_k = Sp\, \hat{A}\, \hat{\sigma}_k$$

$$a_k = \frac{1}{2} Sp\, \hat{A} \hat{\sigma}_k$$

2) $Sp\, \hat{A} = Sp(a_0 I + \vec{a}\hat{\vec{\sigma}}) = 2a_0$

$$a_0 = \frac{1}{2} Sp\, \hat{A}$$

$$a_0 I + a_1 \hat{\sigma}_x + a_2 \hat{\sigma}_y + a_3 \hat{\sigma}_z = 0$$

при $a_0 = a_1 = a_2 = a_3 = 0$

5.8

$\cancel{(\vec a \vec b)}$ $(\vec a \hat{\vec\sigma})^n$

$(\vec a \hat{\vec\sigma})^2 = (a_i \hat\sigma_i)(a_k \hat\sigma_k) =$

$= a_i a_k (\delta_{ik} + i \varepsilon_{ikl} \hat\sigma_l) =$

$= \underset{\overset{\shortparallel}{|\vec a|^2}}{a_i a_k} + i \varepsilon_{ikl} a_i a_k \sigma_l = |\vec a|^2 + \ldots$

$\ldots = (\underset{\shortparallel 1}{\overset{=1}{\varepsilon_{123}}} a_1 a_2 \hat\sigma_z + \underset{\shortparallel 1}{\overset{=-1}{\varepsilon_{213}}} a_2 a_1 \hat\sigma_z + \overset{\shortparallel 1}{\varepsilon_{231}} a_2 a_3 \sigma_x$

$+ \varepsilon_{321} a_3 a_2 \sigma_x + \varepsilon_{132} a_1 a_3 \hat\sigma_y +$

$+ \varepsilon_{312} a_3 a_1 \hat\sigma_y) = |\vec a|^2$

$(\vec a \hat{\vec\sigma})^3 = (\vec a \vec\sigma)^2 (\vec a \vec\sigma) = |\vec a|^2 (\vec a \vec\sigma)$

$(\vec a \vec\sigma)^n = \begin{cases} |a|^n & n = 2k,\ k \in \mathbb{N} \\ |a|^{n-1} (\vec a, \hat{\vec\sigma}) & n = 2k+1,\ k \in \mathbb{N} \end{cases}$

#

$(\hat{\vec\sigma}_1 \hat{\vec\sigma}_2)^n - ?$ $\quad \hat{\vec\sigma}_1 \hat{\vec\sigma}_2$

$\hat{\vec\sigma}_1 \hat{\vec\sigma}_2 = 2(\vec S^2 - \vec S_1^2 - \vec S_2^2)$

$\hat{\vec S} = \hat{\vec S}_1 + \hat{\vec S}_2; \quad \hat{\vec S}^2 = \hat{\vec S}_1^2 + \hat{\vec S}_2^2 + \frac{1}{2} \hat{\vec\sigma}_1 \hat{\vec\sigma}_2$

(5)

$$\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 = 2\left(\hat{\vec{S}}^2 - \hat{\vec{S}}_1^2 - \hat{\vec{S}}_2^2\right) = 2\vec{S}^2 - 3 =$$

$$= \begin{cases} -3, & S=0 \\ 1, & S=1 \end{cases}$$

$$\vec{S}_1 = \frac{1}{2}\vec{\sigma}_1$$

$$(\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 - 1)(\hat{\sigma}_1 \hat{\sigma}_2 + 3) = 0$$

σ_x

$$(\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)^2 = 3 - 2\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2$$

$$(\sigma_1 \sigma_2)^n = A + B\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2$$

$$\begin{cases} \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2 \chi_{S=0} = -3\chi_{S=0} \\ \sigma_1 \sigma_2 \chi_{S=1} = \chi_{S=1} \end{cases}$$

$$\cancel{(A_n + B_n \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)}$$

$$(A_n + B_n \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)\chi_{S=0} = (\hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2)^n \chi_{S=0} =$$

$$= A_n - 3B_n = -3^n$$

$$A_n + B_n = 1$$

$$A_n = 1 - B_n$$

$$1 - B_n - 3B_n = (-3)^n$$

$$B_n = \frac{-(-3)^{n+1}}{4}$$

$$A_n = 1 + \frac{(-3)^{n+1}}{4}$$

$$(\vec{\hat{\sigma}}_1 \vec{\hat{\sigma}}_2)^n = \frac{3}{4}\left(1 - (-3)^{n-1}\right) + \frac{1}{4}\left(1 - (-3)^n\right)\vec{\hat{\sigma}}_1\vec{\sigma}_2$$

$$\cancel{\chi = c\begin{pmatrix} a \\ b \end{pmatrix}}$$

$$\varphi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi = (a\varphi_{1/2} + b\varphi_{-1/2})$$

$$\varphi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$Sp(\chi\chi^*) = 1$$

$$\chi = c\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \varphi_{1/2} & 0 \\ \varphi_{-1/2} & 0 \end{pmatrix}$$

$$\varphi_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi = (a\varphi_{1/2} + b\varphi_{-1/2})c$$

$$\varphi_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$Sp(\chi\chi^*) = 1 = Sp\, c\begin{pmatrix} a \\ b \end{pmatrix} c^*(a^* b^*) =$$

$$= |c|^2(|a|^2 + |b|^2) = 1 \qquad |c| = \frac{1}{\sqrt{|a|^2 + |b|^2}}$$

$\langle \hat{S}_x \rangle = Sp \langle \chi | \hat{S}_x | \chi \rangle =$

$= Sp\, c^* (a^* b^*) \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} c \begin{pmatrix} a \\ b \end{pmatrix} = \frac{|c|^2}{2} \cdot$

$\cdot Sp (a^* b^*) \begin{pmatrix} b \\ a \end{pmatrix} = \frac{|c|^2}{2} (a^* b + \underbrace{b^* a}_{(a^* b)^*}) =$

$= \frac{|c|^2}{2} (Re\, a^* b + i Im\, a^* b + Re\, a^* b^* - i Im (a^* b)) =$

$= \frac{Re\, a^* b}{|a|^2 + |b|^2}$

$\langle \hat{S}_y \rangle = \ldots \langle \hat{S}_z \rangle =$

05.12.12

#...

$S = \frac{1}{2}$,

$\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}} \rightarrow \hat{J}^2 = \hat{L}^2 + \hat{S}^2 + 2 \hat{\vec{L}} \hat{\vec{S}} \Rightarrow \frac{1}{2} \vec{\sigma}$

$\Rightarrow L \sigma = J^2 - L^2 - S^2$

$\langle \vec{L} \vec{\sigma} \rangle$

$\langle SL | \hat{\vec{L}} \hat{\vec{\sigma}} | SL \rangle =$

$|S, L\rangle$

$= \langle \hat{\vec{L}} \hat{\vec{\sigma}} \rangle = |^{\wedge}$

m, S_z

$J = |L - S|, \ldots, L + S$

$L - \frac{1}{2} \qquad L + \frac{1}{2}$

$|SL\rangle = \alpha |SL, L - \frac{1}{2}\rangle +$

$+ \beta |SL, L + \frac{1}{2}\rangle$

$|\alpha|^2 + |\beta|^2 = 1$

$$\hat{A} = J(J+1) - L(L+1) - \frac{1}{2}\left(\frac{1}{2}+1\right) =$$

$$= \begin{cases} \left(L+\frac{1}{2}\right)\left(L+\frac{3}{2}\right) - L(L+1) - \frac{3}{4} = L, & J = L+\frac{1}{2} \\ \left(L-\frac{1}{2}\right)\left(L+\frac{1}{2}\right) - L(L+1) - \frac{3}{4} = -L-1, & J = L-\frac{1}{2} \end{cases}$$

#5.13

$S = \frac{1}{2}$

1. $\hat{P}_{S_z = \pm\frac{1}{2}}$ — ?

$\chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |S_z = \frac{1}{2}>$

$\chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |S_z = -\frac{1}{2}>$

$\hat{P}_{1/2}\chi_{1/2} = \chi_{1/2}$ $\qquad \hat{P}_{-1/2}\chi_{1/2} = 0$

$\hat{P}_{+1/2}\chi_{-1/2} = 0$ $\qquad \hat{P}_{-1/2}\chi_{-1/2} = \chi_{-1/2}$

$$\begin{cases} \hat{P}_{-1/2} = \alpha_-\left(\frac{1}{2} - \hat{S}_z\right) \\ \hat{P}_{+1/2} = \alpha_+\left(\frac{1}{2} + \hat{S}_z\right) \end{cases} \qquad \begin{cases} \alpha_+\left(\frac{1}{2} + \hat{S}_z\right)\chi_{1/2} = \chi_{1/2} = \alpha_+\chi_{1/2} \Rightarrow \alpha_+ = 1 \\ \alpha_-\left(\frac{1}{2} - \hat{S}_z\right)\chi_{-1/2} = \alpha_-\chi_{-1/2} \Rightarrow \alpha_- = 1 \end{cases}$$

$\boxed{\hat{P}_{S_z=\pm\frac{1}{2}} = \frac{1}{2}(1\pm\hat{\sigma}_z) =}$ ombem

$= \frac{1}{2}(1\pm\hat{\vec{\sigma}}\vec{e}_z)$

5.14

$S = \frac{1}{2}$

$\hat{P}_{S_n=\pm\frac{1}{2}}$

$\vec{n}(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$\hat{P}_{S_n=\pm 1/2} = \frac{1}{2}(1\pm\hat{\vec{\sigma}}\vec{n})$

$\chi_{S_n=\frac{1}{2}} = \alpha\chi_{S_z=\frac{1}{2}} + \beta\chi_{S_z=-\frac{1}{2}}$

$\hat{P}_{S_n=\frac{1}{2}}\chi_{S_n=\frac{1}{2}} = \chi_{S_n=\frac{1}{2}}$

$P_{S_n=\frac{1}{2}}\chi_{S_n=-\frac{1}{2}} = 0$

$\chi_{S_z=\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix} = \gamma\chi_{S_n=\frac{1}{2}} + \delta\chi_{S_n=\frac{1}{2}}$

$\Rightarrow \alpha_+ = 1$ $\quad P_{S_n=\frac{1}{2}}\chi_{S_z=\frac{1}{2}} = \gamma\chi_{S_n=\frac{1}{2}}$; $\chi^+_{S_n=\frac{1}{2}}\chi_{S_n=\frac{1}{2}} = 1$

$\Rightarrow \alpha_- = 1$ $\quad \frac{1}{2}(\hat{I}+\hat{\vec{\sigma}}\vec{n})\begin{pmatrix}1\\0\end{pmatrix} = \gamma\begin{pmatrix}1\\0\end{pmatrix}$

$$\vec{\sigma}\vec{n} = \hat{\sigma}_x n_x + \hat{\sigma}_y n_y + \hat{\sigma}_z n_z$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\sin\theta\cos\varphi + \right.$$

$$\left. + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\sin\theta\sin\varphi + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cos\theta\right]\begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$= \gamma\chi_{s_n = \frac{1}{2}}$$

$$\frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & \sin\theta\cos\varphi \\ \sin\theta\cos\varphi & 0 \end{pmatrix} + \right.$$

$$\left. + \begin{pmatrix} 0 & -i\sin\theta\sin\varphi \\ i\sin\theta\sin\varphi & 0 \end{pmatrix} + \begin{pmatrix} \cos\theta & 0 \\ 0 & -\cos\theta \end{pmatrix}\right]\begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$= \gamma\chi_{s_n = \frac{1}{2}}$$

$$\frac{1}{2}\left[\begin{pmatrix} 1+\cos\theta & \sin\theta\cos\varphi - i\sin\theta\sin\varphi \\ \sin\theta\cos\varphi + i\sin\theta\sin\varphi & 1-\cos\theta \end{pmatrix}\right]\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \gamma\chi_{s_n = \frac{1}{2}}$$

$$\cancel{\frac{1}{2}\begin{pmatrix} 1+\cos\theta \\ \sin\theta\cos\varphi + i\sin\theta\sin\varphi \end{pmatrix} = \gamma\chi_{s_n = \frac{1}{2}}}$$

$$\frac{1}{2}\begin{pmatrix} 1+\cos\theta & \sin\theta e^{-i\varphi} \\ \sin\theta e^{i\varphi} & 1-\cos\theta \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$= \frac{1}{2}\begin{pmatrix} 1+\cos\theta \\ \sin\theta e^{i\varphi} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2\cos^2\frac{\theta}{2} \\ 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} e^{i\varphi} \end{pmatrix} =$$

$$= \cos\frac{\theta}{2}\begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\varphi} \end{pmatrix} = \gamma\chi_{s_n = \frac{1}{2}}$$

$$\chi^+_{S_n=\frac{1}{2}}\,\chi_{S_n=\frac12}=1 \qquad \text{"}\chi^+_{n=\frac12}\text{"}\ \text{"}\chi_{n=\frac12}\text{"} \qquad \langle S_{n=\frac12}|S_{n=\frac12}\rangle=1$$

$$\frac{1}{\gamma^2}\cos^2\frac{\theta}{2}\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{-i\varphi}\end{pmatrix}^T\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\varphi}\end{pmatrix}=\cancel{\gamma}\,1$$

$$|\gamma|^2=\cos^2\frac{\theta}{2}\left(\cos\frac{\theta}{2}\ \ \sin\frac{\theta}{2}e^{-i\varphi}\right)\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\varphi}\end{pmatrix}=$$

$$=\cos^2\frac{\theta}{2}\begin{pmatrix}\cos^2\frac{\theta}{2}\\ \sin^2\frac{\theta}{2}\end{pmatrix}=\cos^2\frac{\theta}{2}$$

$$|\gamma|^2=\cos^2\frac{\theta}{2}\begin{pmatrix}\cos^2\frac{\theta}{2}\\ \sin^2\frac{\theta}{2}\end{pmatrix}\Rightarrow$$

$$=|\gamma|^2=\cos^2\frac{\theta}{2}$$

$$\gamma=e^{i\lambda}\cos\frac{\theta}{2}$$

$$\boxed{\chi_{S_n=\frac12}=e^{-i\xi\lambda}\begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}e^{i\varphi}\end{pmatrix}}$$

gauss
$$\boxed{\chi_{S_n=-\frac12}=e^{i\delta}\begin{pmatrix}\sin\frac{\theta}{2}\\ -\cos\frac{\theta}{2}e^{i\varphi}\end{pmatrix}}$$

[redacted]

$$S=\frac12$$

$$|l,m,S_z\rangle=\alpha\,|L\,m\,S_z\ J=L-\tfrac12\rangle+\beta\,|L\,m\,S_z\ J=L+\tfrac12\rangle$$

$$W(J=L-\tfrac12)=W_- \quad ; \quad W(J=L+\tfrac12)=W_+$$

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$$|L, m, S_z\rangle = \alpha\,|L\, m\, S_z\; J = L - \tfrac{1}{2}\rangle +$$
$$+ \beta\,|L\, m\, S_z\; J = L + \tfrac{1}{2}\rangle$$

$$\begin{cases} \hat{P}_{J = L + \frac{1}{2}} = \hat{P}_+ \\ \hat{P}_{J = l - \frac{1}{2}} = \hat{P}_- \end{cases}$$

$$\hat{P}_+ |L + \tfrac{1}{2}\rangle = |L + \tfrac{1}{2}\rangle \rightarrow |^A \quad \hat{P}_- |L + \tfrac{1}{2}\rangle = 0 \Rightarrow |^C$$

$$\hat{P}_+ |L - \tfrac{1}{2}\rangle = 0 \rightarrow |^B \quad \hat{P}_- |L - \tfrac{1}{2}\rangle = |L - \tfrac{1}{2}\rangle$$

$$\langle \hat{\vec{L}}\hat{\vec{\sigma}} \rangle = \begin{cases} L, & J = L + \frac{1}{2} \\ -L-1, & J = L - \frac{1}{2} \end{cases}$$

$$|^B \Rightarrow \hat{P}_+ = \gamma_+ (L + 1 + \hat{\vec{L}}\hat{\vec{\sigma}})$$

$$|^A \longrightarrow \gamma_+ = \frac{1}{2L+1}$$

$$|^C \Rightarrow \hat{P}_- = \gamma_- (L - \hat{\vec{L}}\hat{\vec{\sigma}})$$

$$\gamma_- = \frac{1}{2L+1}$$

$$\boxed{\hat{P}_+ = \frac{L + 1 + \hat{\vec{L}}\hat{\vec{\sigma}}}{2L+1}}$$

$$\boxed{\hat{P}_- = \frac{L - \hat{\vec{L}}\hat{\vec{\sigma}}}{2L+1}}$$

$$\langle l m s_z | \hat{P}_+ | l m s_z \rangle = \left(\alpha^* \langle l m s_z\, J = l - \tfrac{1}{2} | + \right.$$

$$\left. + \beta^* \langle l m s_z \; J = l + \tfrac{1}{2} \right) \cdot \beta | l m s_z \; J = l + \tfrac{1}{2} \rangle =$$

$$\hat{\vec{L}}\hat{\vec{\sigma}} = L_z \sigma_z = 2 L_z S_z$$

$$= |\beta|^2 = W_+ = \langle l m s_z | \frac{l + 1 + \hat{L}\hat{\sigma}}{2l+1} | l m s_z \rangle = 1^*$$

$$\langle l m s_z | \hat{P}_- | l m s_z \rangle = |\alpha|^2 = W_-$$

$$1^* = \frac{l + 1 + \langle \hat{L}\hat{\sigma} \rangle}{2l+1} = \frac{l + 1 + 2 m s_z}{2l+1}$$

$$W_- = \frac{l - \langle L\sigma \rangle}{2l+1} = \frac{1 - 2 m s_z}{2l+1}$$

5.23 — 12.12.12

$$\hat{C}\, \psi_{\alpha\beta} = \psi_{\beta\alpha}$$

$$\psi_{\alpha\beta} \quad \psi_{S S_z} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_\alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix}_\beta$$

$$\left.\begin{aligned} \psi_{11} &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \\ \psi_{10} &= \frac{1}{\sqrt{2}} \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 + \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right) \\ \psi_{1-1} &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 \end{aligned}\right\} S = 1$$

$$\psi_{00} = \frac{1}{\sqrt{2}} \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}_2 - \begin{pmatrix} 0 \\ 1 \end{pmatrix}_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix}_2 \right) \rightarrow S = 0$$

$$\chi_{\alpha\beta} = \frac{1}{2}\chi_{\alpha\beta} + \frac{1}{2}\chi_{\alpha\beta} - \frac{1}{2}\chi_{\beta\alpha} + \frac{1}{2}\chi_{\beta\alpha} =$$

$$= \underbrace{\frac{1}{2}\left(\chi_{\alpha\beta} + \chi_{\beta\alpha}\right)}_{\chi^{(S)}_{\alpha\beta} = \chi^{(S)}_{\beta\alpha}} + \underbrace{\frac{1}{2}\left(\chi_{\alpha\beta} - \chi_{\beta\alpha}\right)}_{\chi^{(A)}_{\alpha\beta} = -\chi^{(A)}_{\beta\alpha}}$$

$$\hat{P}_{S=1} = \frac{1}{4}\left(1 - \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2\right)$$

$$\hat{P}_{S=0} = \frac{1}{4}\left(3 + \hat{\vec{\sigma}}_1 \hat{\vec{\sigma}}_2\right)$$

$$\hat{P}_{S=1}\chi_{\alpha\beta} = \chi^{(S)}_{\alpha\beta} = \frac{1}{2}\left(\chi_{\alpha\beta} + \chi_{\beta\alpha}\right)$$

$$\hat{P}_{S=0}\chi_{\alpha\beta} = \chi^{(A)}_{\alpha\beta} = \frac{1}{2}\left(\chi_{\alpha\beta} - \chi_{\alpha\beta}\right)$$

$$\hat{C}\chi_{\alpha\beta} = \chi_{\beta\alpha} = \frac{1}{2}\left(\chi_{\beta\alpha} + \chi_{\alpha\beta}\right) + \frac{1}{2}\left(\chi_{\beta\alpha} - \chi_{\alpha\beta}\right) =$$

$$= \chi^{(S)}_{\alpha\beta} - \chi^{(A)}_{\alpha\beta} = \hat{P}_{S=1}\chi_{\alpha\beta} - \hat{P}_{S=0}\chi_{\alpha\beta} =$$

$$= \left(\hat{P}_{S=1} - \hat{P}_{S=0}\right)\chi_{\alpha\beta} = \left(\frac{1}{4}\left(1 - \hat{\vec{\sigma}}_1\hat{\vec{\sigma}}_2\right) - \frac{1}{4}\left(3 + \hat{\vec{\sigma}}_1\hat{\vec{\sigma}}_2\right)\right)\chi_{\alpha\beta}$$

$$= \frac{1}{4}\left(-2 - 2\hat{\vec{\sigma}}_1\hat{\vec{\sigma}}_2\right)\chi_{\alpha\beta} = -\frac{1}{2}\left(1 + \hat{\vec{\sigma}}_1\hat{\vec{\sigma}}_2\right)\chi_{\alpha\beta}$$

$$\hat{C} = -\frac{1}{2}\left(1 + \hat{\vec{\sigma}}_1\hat{\vec{\sigma}}_2\right)$$

$$\psi(\vec r, \sigma) \equiv \psi(\xi)$$

$\psi \to e^{i\varphi}\psi$ ← поворот на угол φ

перст

$$\psi_{(Б)}(\xi_1, \xi_2, \dots, \xi_n) = \psi_{(Б)}(\xi_2, \xi_1, \dots, \xi_n)$$

$$\psi_{(Ф)}(\xi_1, \xi_2, \dots, \xi_n) = \psi_{(Ф)}(\xi_2, \xi_1, \dots, \xi_n)$$

~~[illegible]~~

$$\psi_{f_1}(\xi_1), \quad \psi_{f_2}(\xi_2), \quad \psi_{f_3}(\xi_3)$$

1) $f_1 = f_2 = f_3$, $\quad \psi_1 = \psi_{f_1}(\xi_1)\,\psi_{f_1}(\xi_2)\,\psi_{f_1}(\xi_3)$

$$|\psi_{f_i}(\xi_i)|^2 = 1, \quad i = 1, 2, 3$$

2) $f_1 = f_2 \ne f_3$ $\quad \cancel{\psi = (\psi_{f_1}(\xi_1)\,\psi_{f_2}(\xi_2)\,\psi_{f_3}(\xi_3)) + (\psi_{f_1}(\xi_1)\,\psi_{f_3}(\xi_3)\,\psi_{f_2}(\xi_2)) +}$

$+$

$$\psi_2 = \psi_{f_1}(\xi_1)\,\psi_{f_1}(\xi_2)\,\psi_{f_3}(\xi_3) + \psi_{f_1}(\xi_1)\,\psi_{f_3}(\xi_2)\,\psi_{f_1}(\xi_3) + \psi_{f_3}(\xi_1)\,\psi_{f_1}(\xi_2)\,\psi_{f_1}(\xi_3)$$

$$\Psi_2^2 = 1 = |\alpha|^2 \cdot 3$$

$$\alpha = \frac{1}{\sqrt{3}}$$

3) $f_1 \neq f_2 \neq f_3$

$$\Psi_3 = \beta\big(\psi_{f_1}(\xi_1)\psi_{f_2}(\xi_2)\psi_{f_3}(\xi_3) +$$

$$+ \psi_{f_1}(\xi_2)\psi_{f_2}(\xi_1)\psi_{f_3}(\xi_3) +$$

$$+ \psi_{f_1}(\xi_3)\psi_{f_2}(\xi_2)\psi_{f_3}(\xi_1) +$$

$$+ \psi_{f_1}(\xi_1)\psi_{f_2}(\xi_3)\psi_{f_3}(\xi_2) +$$

$$+ \psi_{f_1}(\xi_3)\psi_{f_2}(\xi_1)\psi_{f_3}(\xi_2) +$$

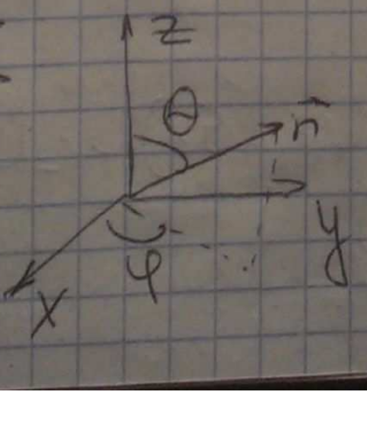
$$+ \psi_{f_1}(\xi_2)\psi_{f_2}(\xi_3)\psi_{f_3}(\xi_1)\big) \Rightarrow$$

$$\Rightarrow |\Psi_3|^2 = 1 = 6|\beta|^2 \qquad \beta = \frac{1}{\sqrt{6}}$$

5.12

$$\psi = e^{i\gamma}\begin{pmatrix}\cos\alpha \\ e^{i\beta}\sin\alpha\end{pmatrix} \qquad s = \frac{1}{2}$$

$\hat{\vec{e}}$



$$\hat{S}_n = \hat{\vec{S}}\vec{n}$$

$$\vec{n}(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

$$\hat{\vec{S}} = \frac{1}{2}\Big(\underbrace{\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}}_{S_x}, \underbrace{\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}}_{S_y}, \underbrace{\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}}_{S_z}\Big)$$

$$\hat{S}_n = \vec{\hat{S}}\vec{n} = \hat{S}_x n_x + \hat{S}_y n_y + \hat{S}_z n_z$$

$$\hat{S}_n = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sin\theta\cos\varphi + \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sin\theta\sin\varphi +$$

$$+ \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cos\theta = |^{A}$$

$$\hat{S}_n \psi = \frac{1}{2}\psi \qquad \theta - ? \quad \varphi - ?$$

$$^{A}| = \frac{1}{2}\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta\, e^{i\varphi} & -\cos\theta \end{pmatrix}$$

$$\hat{S}_n \psi = \frac{1}{2}\psi$$

$$\cancel{\frac{1}{2}e^{i\gamma}\begin{pmatrix} \cos\theta & \sin\theta\, e^{i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin \end{pmatrix}}$$

$$\frac{1}{2}e^{i\gamma}\begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\varphi} \\ \sin\theta e^{i\varphi} & -\cos\theta \end{pmatrix}\begin{pmatrix} \cos\alpha \\ e^{i\beta}\sin\alpha \end{pmatrix} = \frac{1}{2}e^{i\gamma}\begin{pmatrix} \cos\alpha \\ e^{i\beta}\sin\alpha \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta\cos\alpha + \sin\theta\sin\alpha\, e^{i(\beta-\varphi)} \\ \sin\theta\cos\alpha\, e^{i\varphi} - \cos\theta\sin\alpha\, e^{i\beta} \end{pmatrix} = \begin{pmatrix} \cos\alpha \\ e^{i\beta}\sin\alpha \end{pmatrix}$$

$$\begin{cases} \cos\theta\cos\alpha + \sin\theta\sin\alpha\, e^{i(\beta-\varphi)} = \cos\alpha \\ \sin\theta\cos\alpha\, e^{i\varphi} - \cos\theta\sin\alpha\, e^{i\beta} = e^{i\beta}\sin\alpha \end{cases}$$

$$\begin{cases} \sin\theta\sin\alpha\, e^{i(\beta-\varphi)} = \cos\alpha(1-\cos\theta) \\ \sin\theta\cos\alpha\, e^{i(\varphi-\beta)} = \sin\alpha(1+\cos\theta) \end{cases}$$

$$\boxed{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}$$

$$\begin{cases} \sin\theta \sin\alpha\, e^{i(\beta-\varphi)} = 2\cos\alpha \sin^2\frac{\theta}{2} \\ \sin\theta \cos\alpha\, e^{i(\varphi-\beta)} = 2\sin\alpha \cos^2\frac{\theta}{2} \end{cases}$$

$$\begin{cases} \sin\theta\, \mathrm{tg}\,\alpha\, e^{i(\beta-\varphi)} = 2\sin^2\frac{\theta}{2} \\ \sin\theta\, e^{i(\varphi-\beta)} = 2\,\mathrm{tg}\,\alpha \cos^2\frac{\theta}{2} \end{cases}$$

$$\begin{cases} \cos\frac{\theta}{2}\, \mathrm{tg}\,\alpha\, e^{i(\beta-\varphi)} = \sin\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, e^{i(\varphi-\beta)} = \mathrm{tg}\,\alpha \cos\frac{\theta}{2} \end{cases}$$

$$\begin{cases} \mathrm{tg}\,\alpha\, e^{i(\beta-\varphi)} = \mathrm{tg}\,\frac{\theta}{2} \\ \mathrm{tg}\,\frac{\theta}{2} = e^{i(\beta-\varphi)}\, \mathrm{tg}\,\alpha \end{cases}$$

$\frac{\theta}{2} = \alpha + \pi n$ $\qquad$ $0 \le \alpha \le \frac{\pi}{2} \Rightarrow \boxed{\theta = 2\alpha}$

$\beta - \varphi = \pi k$ $\qquad$ $0 \le \beta \le 2\pi \Rightarrow \boxed{\varphi = \beta}$

1) $\theta = 0$, $\varphi = 0$

$\psi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \psi_{1/2}$

2) $\theta = \pi$, $\varphi = 0$ $\qquad$ $\psi = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \psi_{-1/2}$