Доза #1

#70

$\rho = \rho_0 \cos(\alpha x)\cos(\beta y)\cos(\gamma z)$ $\varphi - ?$

Ищем решение в виде: $\varphi = \varphi_0 \cos\alpha x \cos\beta y \cos\gamma z$

$\Delta\varphi = -4\pi\rho \Rightarrow$

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial^2\varphi}{\partial z^2} = -4\pi\rho_0 \cos(\alpha x)\cos(\beta y)\cos(\gamma z)$$

$$\varphi_0\left(-\alpha^2\cos(\alpha x)\cos(\beta y)\cos(\gamma z) - \beta^2\cos - \right.$$

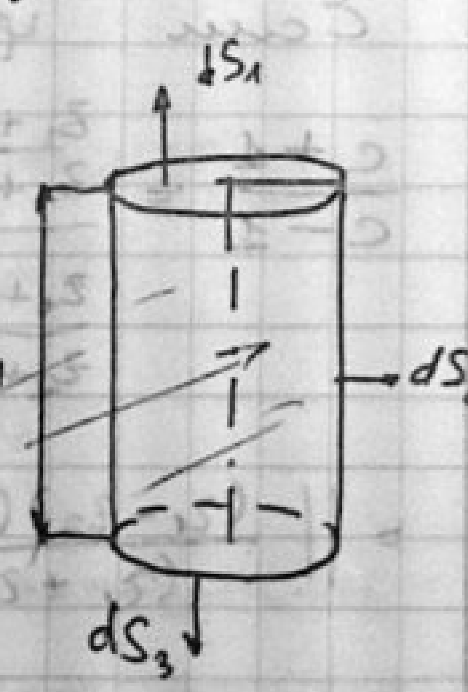
$$\left. - \gamma^2 \cos\right) = -4\pi\rho_0 \cos\alpha x \cos\beta y \cos\gamma z$$

$$\Rightarrow \varphi_0(\alpha^2 + \beta^2 + \gamma^2) = 4\pi\rho_0$$

$$\Rightarrow \varphi_0 = \frac{4\pi\rho_0}{\alpha^2+\beta^2+\gamma^2}$$

$$\varphi = \frac{4\pi\rho_0}{\alpha^2+\beta^2+\gamma^2}\cos(\alpha x)\cos(\beta y)\cos(\gamma z)$$

#73



$\varphi - ?$

$E - ?$

$$\varphi \neq \varphi(\alpha) \quad \bar{E} \neq \bar{E}(\alpha)$$

$$\Rightarrow \bar{E} = E(r)\vec{e}_r$$

$$(d\vec{S} \cdot \bar{e}_z) = 0$$

т. Гаусса: $\oint \vec{E}\, d\vec{S} = 4\pi\rho \Rightarrow$

$$\Rightarrow E\int dS_2 = E \cdot 2\pi r h = 4\pi\rho$$

$$\frac{q}{h} = \varkappa \qquad \vec{E} = \frac{d\varkappa}{r}\vec{e_r}$$

$$\varphi = -\int \vec{E}\,d\vec{r} = -2\varkappa \ln r + \varphi$$

При $r = 1$; $\varphi = 0 \Rightarrow \varphi_0 = 0$

$$\varphi = -2\varkappa \ln r \ ; \qquad \vec{E} = \frac{2\varkappa}{r}\vec{e_r}$$

#75.

Из №74

$$\varphi(x,y,z) = -\frac{q}{2a}\ln\left|\frac{z-a+\sqrt{(z-a)^2+x^2+y^2}}{z+a+\sqrt{(z+a)^2+x^2+y^2}}\right|$$

Сделаем замену: $z_1 = z+a$; $z_2 = z-a$.

$$r_{1,2} = \sqrt{z_{1,2}^2+x^2+y^2}$$

Если $\varphi = const \Rightarrow \frac{z_1+r_1}{z_2+r_2} = const$

$$\frac{c+1}{c-1} = \frac{\frac{z_1+r_1}{z_2+r_2}+1}{\frac{z_1+r_1}{z_2+r_2}-1} = \left\{\begin{matrix} z_1+z_2 = 2z \\ z_1-z_2 = 2a \end{matrix}\right\} = \frac{2z+r_1+r_2}{2a+r_1+r_2} =$$

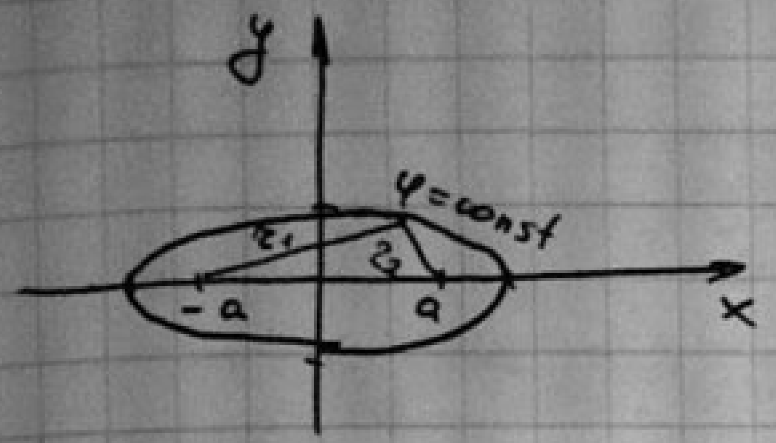
$$= \left\| \frac{(r_1-r_2)(r_1+r_2)}{(r_1+r_2)} = \frac{r_1^2-r_2^2}{r_1+r_2} = \frac{z_1^2-z_2^2}{r_1+r_2} = \frac{(z_1-z_2)(z_1+z_2)}{r_1+r_2} = \right.$$

$$= \frac{2a z}{r_1+r_2} \left\| = \frac{2z+r_1+r_2}{2a+\frac{4az}{r_1+r_2}} = \frac{2z+r_1+r_2}{\frac{2a}{r_1+r_2}(r_1+r_2+2z)} = \frac{r_1+r_2}{2a}\right.$$

$$r_1+r_2 = 2a\frac{c+1}{c-1} = const$$

При $r_1 + r_2 = C$, $\Rightarrow$ эллипс с фокусами

$z = \pm a$.



78 Дано: R, ρ, R_1, $a + R_1 < R$.

Найти: E (электрическое поле) - ?

Заменим шар с пустотой на шар без пустоты с ρ^+ в котором есть шар с ρ^-, радиуса R на расстоянии a от центра

$\operatorname{div} \vec{E} = 4\pi\rho$. из симметрии: $\vec{E} = \vec{E}(R_1)$

$\vec{E} = E(R_1)\vec{e_r}$

$$\operatorname{div} \vec{E} = \frac{1}{R_1^2}\frac{\partial}{\partial R_1}(R_1^2 E) = 4\pi\rho - \Delta\varphi.$$

$$R_1^2 E = \int 4\pi\rho R_1^2\, dR_1 = \frac{4\pi}{3}\rho R_1^3$$

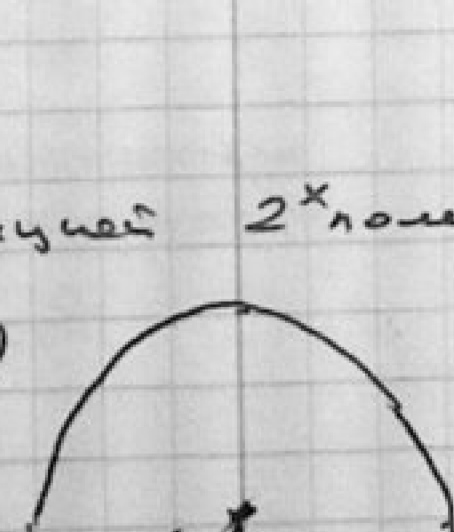
$$E = \frac{4\pi}{3}\rho R_1 \qquad \vec{E} = \frac{4\pi}{3}\rho \vec{R_1}$$

поле в пустоте есть суперпозицией $2^х$ полей

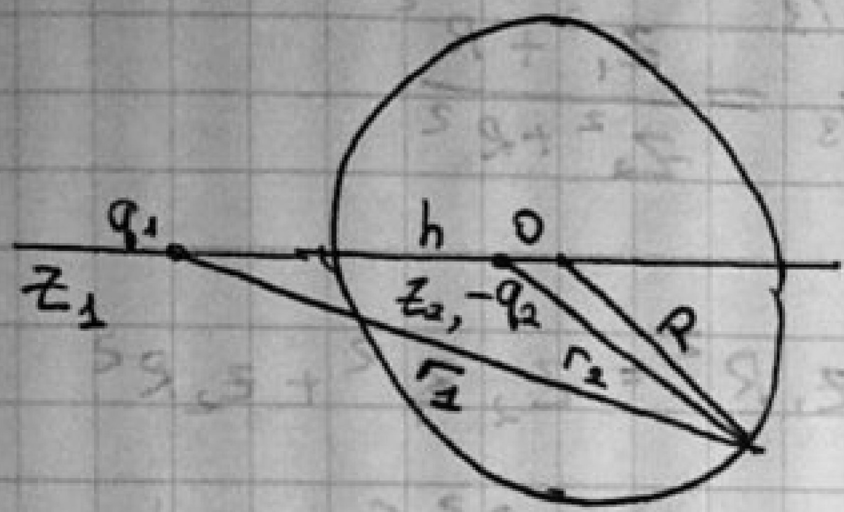
$$\vec{E_1} = \frac{4\pi}{3}\rho R, \qquad \vec{E_2} = \frac{4\pi}{3}(-\rho)(\vec{R} - \vec{a})$$

$$\vec{E}_{пуст} = \frac{4\pi}{3}\rho\vec{R} - \frac{4\pi}{3}\rho(\vec{R} - \vec{a}) = \frac{4\pi}{3}\rho\vec{a}$$

$$\vec{E}_{пуст} = \frac{4\pi}{3}\rho\vec{a}$$



Доза #2. #118 #147



$$\varphi = \frac{q_1}{r_1} - \frac{q_2}{r_2}$$, не зависит от θ

$$r_{1,2} = \sqrt{z_{1,2}^2 + R^2 - 2z_{1,2} R\cos\theta}$$

$$\frac{\partial\varphi}{\partial\theta} = 0 \Rightarrow -\frac{q_1}{2r_1^3}(2z_1 R\sin\theta) + \frac{q_2}{2r_2^3}(2z_2 R\sin\theta)$$

$$\Rightarrow q_1 z_1 r_2^3 = q_2 z_2 r_1^3 \quad \Big| \wedge 2/3$$

$$q_1^{2/3} z_1^{2/3} (z_2^2 + R^2 - 2z_2 R\cos\theta) =$$
$$= q_2^{2/3} z_2^{2/3} (z_1^2 + R^2 - 2z_1 R\cos\theta)$$

1) $q_1^{2/3} z_1^{2/3} z_2 R = q_1^{2/3} z_2^{2/3} z_1 R$

$$\frac{q_1^{2/3}}{q_2^{2/3}} = \frac{z_1^{1/3}}{z_2^{1/3}} \longrightarrow \boxed{\frac{q_1^2}{q_2^2} = \frac{z_1}{z_2}}$$ Положение центра

2) $q_1^{2/3} z_1^{2/3} (z_2^2 + R^2) = q_2^{2/3} z_2^{2/3} (z_1^2 + R^2)$

$$\frac{q_1^{2/3}}{q_2^{2/3}} \frac{z_1^{2/3}}{z_2^{2/3}} = \frac{z_1^2 + R^2}{z_2^2 + R^2}$$

$$\Rightarrow \quad z_2^2 z_1 + z_1 R^2 = z_2 z_1^2 + z_2 R^2$$

$$z_1 z_2 (z_2 - z_1) = R^2 (z_2 - z_1)$$

$$R^2 = z_1 z_2 \to \text{радиус}$$

Проверим потенциал

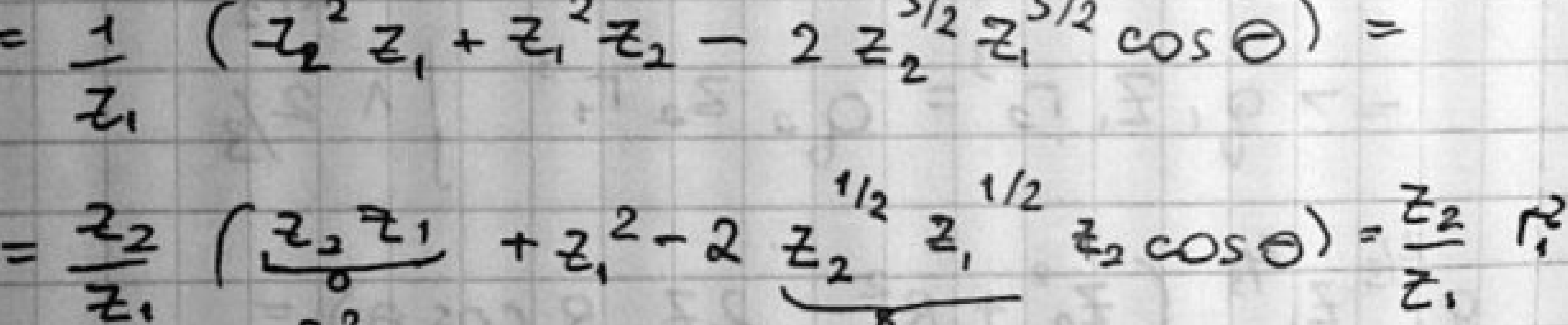
$$r_2^2 = z_1^2 + R^2 - 2z_2 R\cos\theta = (z_2^2 + z_1 z_2 - 2z_2\sqrt{z_1 z_2}\cos\theta) =$$

$$= \frac{1}{z_1}\left(z_2^2 z_1 + z_1^2 z_2 - 2 z_2^{3/2} z_1^{3/2}\cos\theta\right) =$$

$$= \frac{z_2}{z_1}\left(\underbrace{z_2 z_1}_{R^2} + z_1^2 - 2\underbrace{z_2^{1/2} z_1^{1/2}}_{R} z_2\cos\theta\right) = \frac{z_2}{z_1} r_1^2$$

$$q_2 = q_1\sqrt{\frac{z_2}{z_1}} \qquad \frac{1}{r_2} = \frac{1}{r_1}\sqrt{\frac{z_1}{z_2}}$$

$$\varphi = \frac{q_1}{r_1} - \frac{q_1\sqrt{z_1}R'}{r_1}\sqrt{\frac{z_1}{z_2}} = 0$$

#147. 

$$E = \sum_{i=1}^{4} \frac{q_i}{r_i^3}\vec{r}_i$$

Доза # 3.

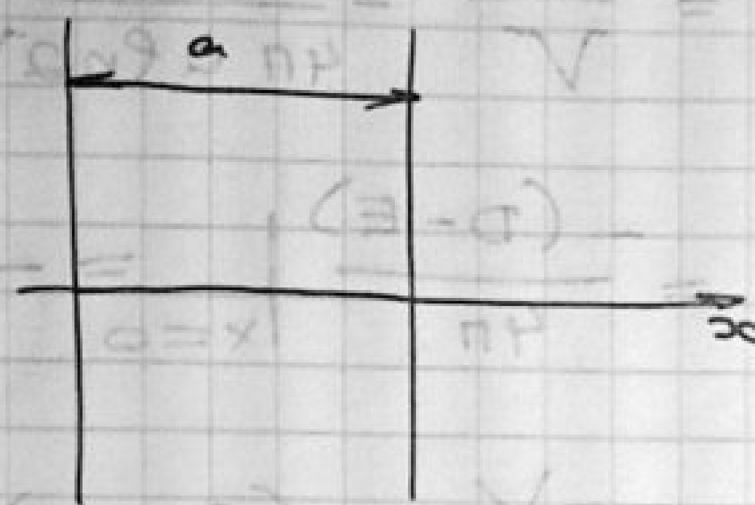
135.

$\varepsilon = \varepsilon_0 \dfrac{x+a}{a}$

V

$\sigma_{св}$ - ?

c - ?



$\operatorname{div} \vec{E} = 0$

$$\frac{d}{dx}\left(\varepsilon \frac{d\varphi}{dx}\right) = 0 \Rightarrow \varepsilon \frac{d\varphi}{dx} = c_1$$

$$\frac{d\varphi}{dx} = \frac{a c_1}{\varepsilon_0 (x+a)}; \quad \varphi = \int^{x} \frac{a}{\varepsilon_0} \frac{c_1}{x'+a}\, dx' = \frac{a}{\varepsilon_0} c_1 \ln \frac{x+a}{a}$$

$$\varphi(a) - \varphi_0(a) = V \Rightarrow$$

$$\frac{a}{\varepsilon_0} c_1 \ln 2 = V$$

$$\Rightarrow c_1 = -\frac{\varepsilon_0 V}{a \ln 2}$$

$$E = -\frac{d\varphi}{dx} = \frac{a}{\varepsilon_0} \frac{\varepsilon_0}{a} \frac{V}{\ln 2} \frac{1}{x+a}\, \varepsilon$$

$$D = \varepsilon E = \cancel{\frac{V}{\ln 2} \frac{\varepsilon_0}{dx}} \quad \frac{V}{\ln 2} \frac{\varepsilon}{a+x} = \frac{\varepsilon_0 V}{a \ln 2} \qquad D = \varepsilon E$$

$$\sigma = \frac{D_n}{4\pi}\Big|_{x=0} = \frac{V \varepsilon_0}{4\pi a \ln 2}$$

$$c = \frac{\sigma S}{V} = \frac{V \varepsilon_0 S}{4\pi a \ln 2 V} = \frac{\varepsilon_0 S}{4\pi a \ln 2}$$

$$\sigma_{cb} = -\frac{(D-E)}{4\pi}\Big|_{x=0} = -\left(\frac{V\varepsilon_0}{a\ln 2} - \frac{V}{a\ln 2}\right)\frac{1}{4\pi} =$$

$$= -\frac{V}{4\pi a \ln 2}(\varepsilon_0 - 1) = -\frac{V\varepsilon_0}{4\pi a \ln 2}\left(1 - \frac{1}{\varepsilon_0}\right)$$

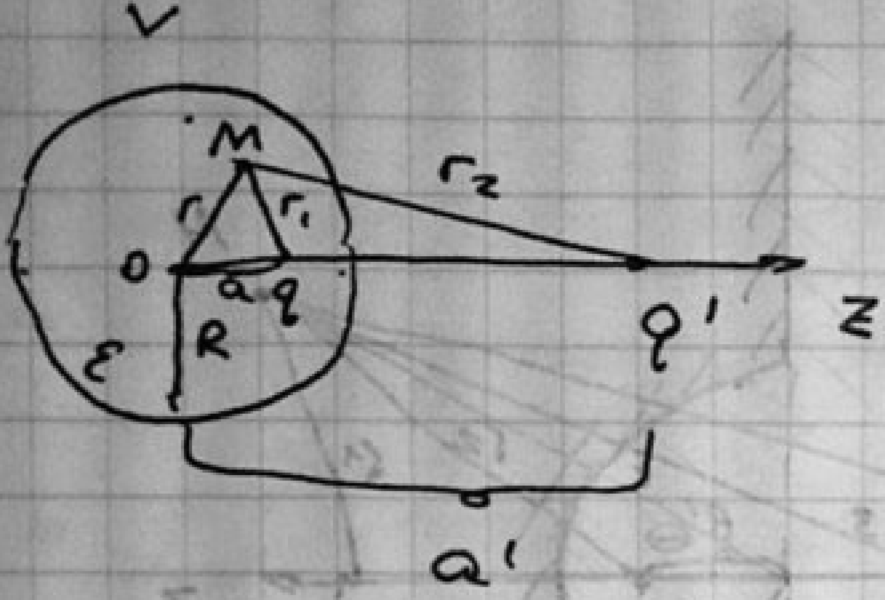
$$\sigma_{cb} = -\frac{V}{4\pi a \ln 2}(\varepsilon_0 - 1) = -\frac{V\varepsilon_0}{4\pi a \ln 2}\left(1 - \frac{1}{\varepsilon_0}\right)$$

$$\sigma_{cb} = \frac{D-E}{4\pi}\Big|_{x=a} = \left(\frac{V\varepsilon_0}{a\ln 2} - \frac{V}{2a\ln 2}\right)\frac{1}{4\pi} =$$

$$= \frac{V}{4\pi a \ln 2}\left(\varepsilon_0 - \frac{1}{2}\right) = \frac{V\varepsilon_0}{4\pi a \ln 2}\left(1 - \frac{1}{2\varepsilon_0}\right)$$

Доза #4.

154



$$\begin{cases} \left(\frac{q}{q'}\right)^2 = \frac{a}{a'} \\ aa' = R^2 \end{cases} \Rightarrow q' = q\frac{R}{a}$$

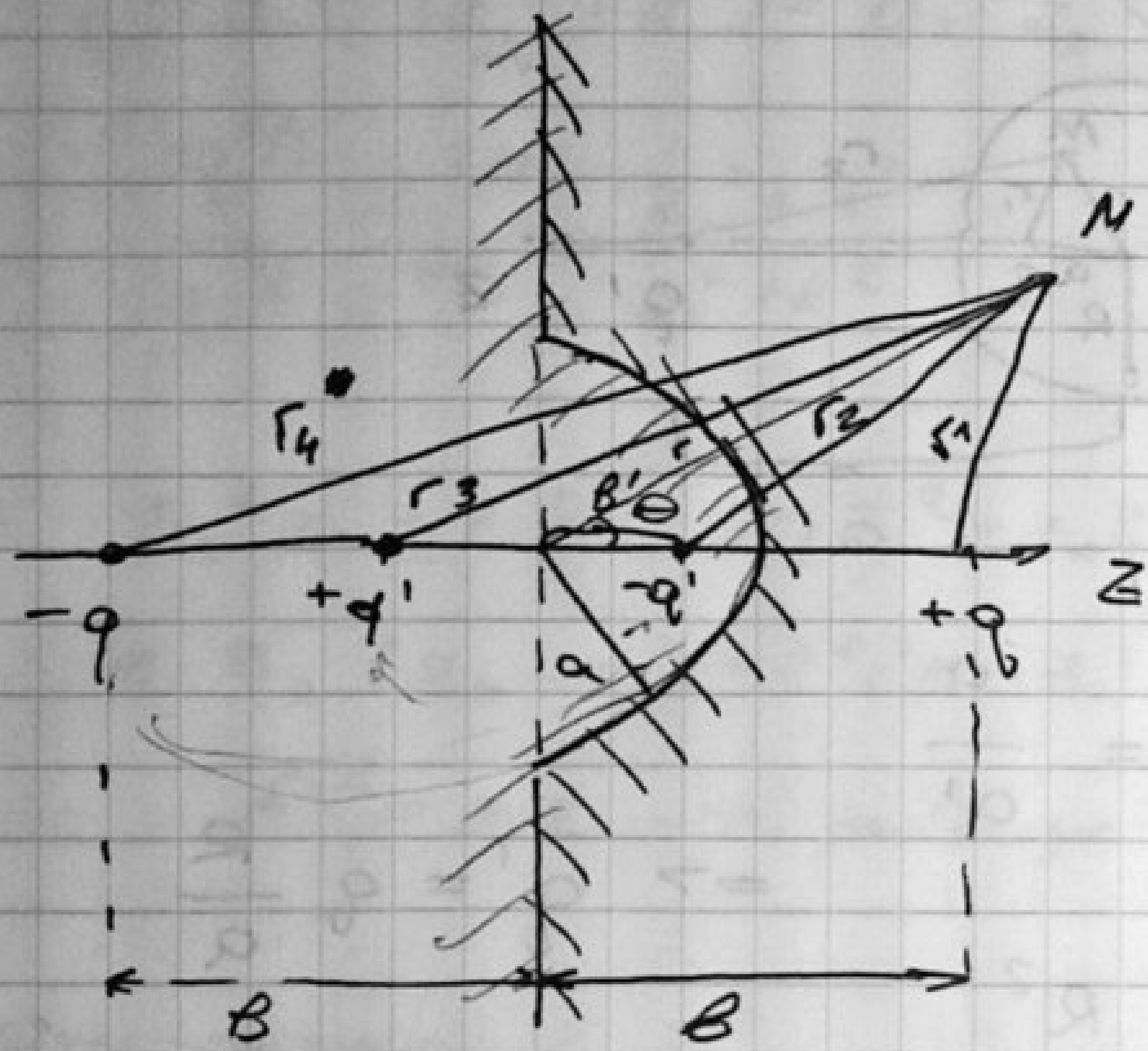
г. у $\varphi|_s = V$

$$\varphi = \frac{q}{\varepsilon r_1} - \frac{q'}{\varepsilon r_2} + c$$

$$\frac{q}{\varepsilon r_1} - \frac{q'}{\varepsilon r_2}\Big|_s = 0 \Rightarrow c = V$$

$$\varphi = \frac{q}{\varepsilon r_1} - \frac{q'}{\varepsilon r_2} + V$$

155



$$q' = q\frac{a}{b} \qquad b' = \frac{a^2}{b}$$

$$\varphi_M = \frac{q}{r_1} - \frac{q'}{r_2} + \frac{q'}{r_3} - \frac{q}{r_4}$$

$$r_1 = (r^2 + b^2 - 2rb\cos\theta)^{1/2}$$

$$r_2 = (r^2 + b'^2 - 2rb'\cos\theta)^{1/2}$$

$$r_3 = (r^2 + b'^2 + 2rb'\cos\theta)^{1/2}$$

$$r_4 = (r^2 + b^2 + 2rb\cos\theta)^{1/2}$$

$$\sigma = -\frac{1}{4\pi}\left.\frac{\partial\varphi}{\partial r}\right|_{r=a}$$

$$\frac{\partial\varphi}{\partial r} = -\left(\frac{q}{r_1^3}(r - b\cos\theta) - \frac{q'}{r_2^3}(r - b'\cos\theta) + \right.$$

$$+\frac{q'}{r_3^3}(r+b'\cos\theta)+\frac{q}{r_4^3}(r+b\cos\theta))$$

$$r_2\Big|_{r=a}=(a^2+b'^2-2ab'\cos\theta)^{1/2}=$$

$$=(bb'+b'^2-2\sqrt{bb'}\,b'\cos\theta)^{1/2}=$$

$$=\sqrt{\frac{b'}{b}}(b^2+b'b-2\sqrt{bb'}\,b\cos\theta)^{1/2}=$$

$$=\sqrt{\frac{b'}{b}}(b^2+a^2-2ab\cos\theta)^{1/2}=$$

$$=\left(\frac{b'}{b}\right)^{1/2}r_1\Big|_{r=a}=\frac{a}{b}r_1$$

$$r_3\Big|_{r=a}=(a^2+b'^2+2ab'\cos\theta)^{1/2}=$$

$$(bb'+b'^2+2\sqrt{bb'}\,b'\cos\theta)=$$

$$=\sqrt{\frac{b'}{b}}(b^2+b'b+2b\sqrt{bb'}\cos\theta)=$$

$$=\sqrt{\frac{b'}{b}}\,r_4\Big|_{r=a}=\frac{a}{b}r_4$$

$$\sigma=-\frac{1}{4\pi}\left[\frac{q}{r_1^3}(a-b\cos\theta)-\frac{q\frac{a}{b}}{(r_1\frac{a}{b})^3}(a-\frac{a^2}{b}\cos\theta)+\right.$$

$$\left.+\frac{q(\frac{a}{b})}{(\frac{a}{b}r_4)^3}(a+\frac{a^3}{b}\cos\theta)-\frac{q}{r_4^3}(a+b\cos\theta)\right]=$$

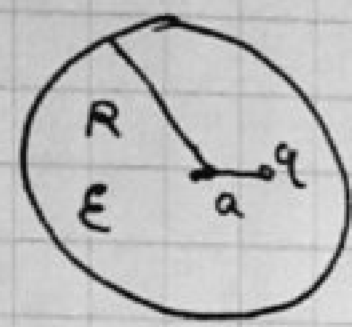
$$=\frac{q}{4\pi}\left(\frac{a-b\cos\theta-b^2/a+b\cos\theta}{r_1^3}+\right.$$

$$\left.+\frac{b^2/a+b\cos\theta-a-b\cos\theta}{r_4^3}\right)=-\frac{q}{4\pi}\left(\frac{b^2}{a}-a\right)\left(\frac{1}{r_1^3}-\frac{1}{r_4^3}\right)$$

Дода #5

154, 158

154:



$E-?$

$$\varphi = \varphi_{ш} + \varphi_{т.з.}$$

$$\varphi_{т.з} = \frac{q}{\varepsilon}\sum_{n=0}^{+\infty}\frac{a^n}{r^{n+1}} P_n(\cos\theta); \quad r \geqslant a$$

$$\varphi_{ш} = \sum_{n=0}^{+\infty} A_n r^n P_n(\cos\theta)$$

$$V = \sum_{n=0}^{+\infty}\left(A_n R^n + \frac{q}{\varepsilon}\frac{a^n}{R^{n+1}}\right) P_n(\cos\theta)$$

$n=0$:

$$A_0 + \frac{q}{\varepsilon}\frac{1}{R} = V \Rightarrow A_0 = V - \frac{q}{\varepsilon R}$$

$$A_n R^n + \frac{q}{\varepsilon}\frac{a^n}{R^{n+1}} = 0 \Rightarrow A_n = -\frac{q}{\varepsilon}\frac{a^n}{R^{2n+1}}$$

$$\varphi = V - \frac{q}{\varepsilon R} + \sum_{n=1}^{+\infty}\left(-\frac{q}{\varepsilon}\right)\frac{a^n}{R^{2n+1}} r^n P_n(\cos\theta) +$$

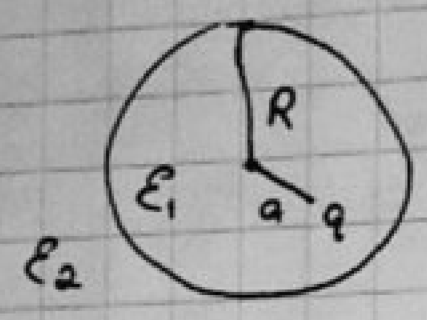
$$+ \frac{q}{\varepsilon}\sum_{n=0}^{+\infty}\frac{a^n}{r^{n+1}} P_n(\cos\theta) = V - \frac{q}{\varepsilon R} + \frac{q}{\varepsilon r} +$$

$$+ \frac{q}{\varepsilon}\sum_{n=1}^{+\infty}\left(\frac{a^n}{r^{n+1}} - \frac{a^n r^n}{R^{2n+1}}\right) P_n(\cos\theta) =$$

$$= V + \frac{q}{\varepsilon}\left(\frac{1}{r} - \frac{1}{R}\right) + \frac{q}{\varepsilon}\sum_{n=1}^{+\infty}\left(\frac{a^n}{r^{n+1}} - \frac{a^n r^n}{R^{2n+1}}\right) P_n(\cos\theta) =$$

$$= V + \frac{q}{\varepsilon} \sum_{n=0}^{+\infty} \left(\frac{a^n}{r^{n+1}} - \frac{a^n r^n}{R^{2n+1}} \right) P_n(\cos\theta)$$

\# 15B.



$$\varphi_1 = \varphi_{шара} + \varphi_q$$

$$\varphi_2 = \varphi_{шара}$$

$$\varphi_1 = \sum_{n=0}^{+\infty} A_n r^n P_n(\cos\theta) + \frac{q}{\varepsilon_1} \sum_{n=0}^{+\infty} \frac{a^n}{r^{n+1}} P_n(\cos\theta)$$

$$\varphi_2 = \sum_{n=0}^{+\infty} \frac{B_n}{r^{n+1}} P_n(\cos\theta)$$

$$\begin{cases} \varphi_1 \big|_{r=R} = \varphi_2 \big|_{r=R} & \varphi_1 = \varphi_2 \\ \varepsilon_1 \varphi_1' \big|_{r=R} = \varepsilon_2 \varphi_2' \big|_{r=R} & D_{1n} = D_{2n} \end{cases}$$

$$\varphi_1 = \varphi_2 \big|_{r=R} \Rightarrow$$

$$A_n R^n + \frac{q}{\varepsilon_1} \frac{a^n}{R^{n+1}} = \frac{B_n}{R^{n+1}}$$

$$\varepsilon_1 \varphi_1' = \varepsilon_2 \varphi_2' \big|_{r=R} \Rightarrow$$

$$\varepsilon_1 A_n R^{n-1} - q \frac{a^n (n-1)}{R^{n+2}} = - \frac{\varepsilon_2 (n+1) B_n}{R^{n+2}} \Bigg| \frac{R}{\varepsilon_2 (n+1)}$$

$$\begin{cases} A_n R^n + \dfrac{q}{\varepsilon_1} \dfrac{a^n}{R^{n+1}} = \dfrac{B_n}{R^{n+1}} \\ \dfrac{\varepsilon_1 n}{\varepsilon_2 (n+1)} A_n R^n - \dfrac{q}{\varepsilon_2} \dfrac{a^n}{R^{n+1}} = - \dfrac{B_n}{R^{n+1}} \end{cases}$$

$$A_n R^n \left[1 + \frac{\varepsilon_1 n}{\varepsilon_2 (n+1)} \right] = \frac{q a^n}{R^{n+1}} \left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1} \right)$$

$$A_n = \frac{q a^m}{R^{2n+1}} \left[\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1} \right] \left[\frac{\varepsilon_2 (n+1)}{\varepsilon_2 (n+1) + \varepsilon_1 n} \right]$$

$$B_n = q a^m \left[\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1} \right] \left[\frac{\varepsilon_2 (n+1)}{\varepsilon_2 (n+1) + \varepsilon_1 n} \right] + \frac{q a^n}{\varepsilon_1}$$

$$\varphi_1 = \sum_{n=0}^{+\infty} \left(\frac{q a^n r^n}{R^{2n+1}} \left[\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1} \right] \left[\frac{\varepsilon_2 (n+1)}{\varepsilon_2 (n+1) + \varepsilon_1 n} \right] \right) P_n(\cos\theta) +$$

$$+ \frac{q}{\varepsilon_1} \sum_{n=0}^{+\infty} \frac{a^n}{r^{n+1}} P_n(\cos\theta)$$

$$a = 0 \longrightarrow n = 0$$

$$\varphi_1 = \frac{q}{R} \left[\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1} \right] \left(\frac{\varepsilon_1}{\varepsilon_2} \right) + \frac{q}{\varepsilon_1 r}$$

$$\varphi_2 = \frac{q}{r} \left(\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - \frac{1}{\varepsilon_1} \right) \right) = \frac{q}{\varepsilon_2 r}$$

$$\varphi_2 = \sum_{n=0}^{+\infty} \frac{q a^n}{r^{n+1}} \left(\frac{1}{\varepsilon_2} \frac{\varepsilon_2 (n+1)}{\varepsilon_2 (n+1) + \varepsilon_1 n} \right) P_n \cos\theta$$

?

Доза #7 (#174).

#174.

$a < R_2 - R_1$

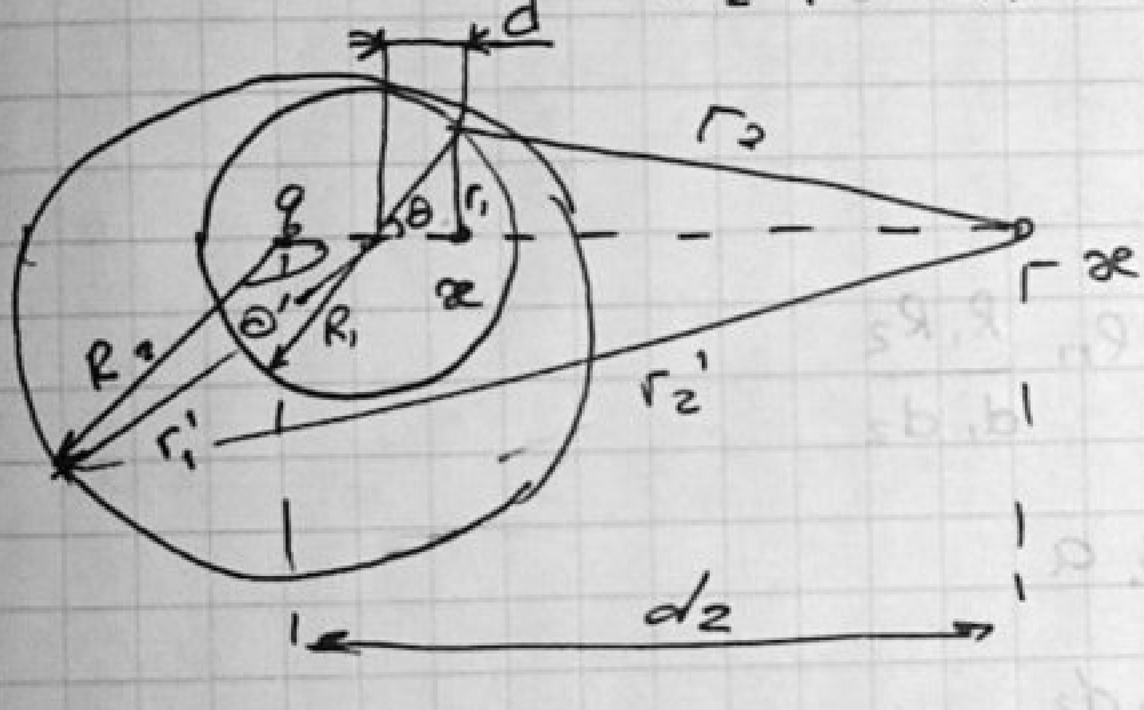
$R_2 > R_1$

$c - ?$

$$c = \frac{q}{|\varphi_2 - \varphi_1|}$$

Заменим цилиндр двумя нитями.

$x_1 x_2 = R_2 \quad \to \quad \begin{cases} d_1 (d_2 - a) = R_1^2 \\ d_2 (d + d_1) = R_2^2 \end{cases}$



внешн.

$\varphi^{(1)}$ – потенциал на $1^{\text{й}}$ обкладке.

$\varphi^{(1)} = -2\varkappa \ln r_1 + 2\varkappa \ln r_2$

$$\begin{cases} r_1^2 = R_1^2 + d^2 - 2R_1 d_1 \cos\theta \\ r_2^2 = R_1^2 + (d_2 - a)^2 - 2R_1 (d_2 - a)\cos\theta \end{cases}$$

$$d_2 - a = \frac{R_1^2}{d_1}$$

$$r_2^2 = R_1^2 + \frac{R_1^4}{d_1^2} - 2R_1 \frac{R_1^2}{d_1}\cos\theta = \frac{R_1^2}{d_1^2} r_1^2$$

$\varphi^{(2)}$ – потенциал внутр. обкладки.

$$\varphi^{(2)} = 2\varkappa \ln \frac{r_2}{r_1} = 2\varkappa \ln \frac{R_1}{d_1}$$

$$\begin{cases} r_2'^2 = R_2^2 + d_2^2 - 2R_2 d_2 \cos\theta' \\ r_1'^2 = R_2^2 + (a+d_1)^2 - 2R_2(a+d_1)\cos\theta' \end{cases}$$

$$(a+d_1)^2 = \frac{R_2^4}{d_2}$$

$$r_1'^2 = R_2^2 + \frac{R_2^4}{d_2^2} - 2R_2 \frac{R_2^2}{d_2}\cos\theta'$$

$$\varphi^{(2)} = 2\varkappa \ln \frac{d_1}{R_2}$$

$$\varphi^{(1)} - \varphi^{(2)} = 2\varkappa \ln \frac{R_1 R_2}{d_1 d_2}$$

$$\begin{cases} d_1 d_2 = R_1^2 + d_1 a \\ d_1 d_2 = R_2^2 - a d_2 \end{cases}$$

$$d_1\left(\frac{R_2^2}{(a+d_1)} - a\right) = R_1^2$$

$$\frac{d_1 R_2^2 - d_1 a^2 - d_1^2 a}{a+d_1} = R_1^2$$

$$d_1 R_2^2 - d_1 a^2 - d_1^2 a - R_1^2 d_2 - R_1^2 a = 0$$

$$d_1^2 + \frac{(R_2^2 - R_1^2 - a^2)d_1 - R_1^2}{a} = 0$$

$$d_1 = \frac{-(R_2^2 - R_1^2 - a^2) \pm \sqrt{(R_2^2 - R_1^2 - a^2)^2 + 4R_1^2}}{2a}$$

$$d_1^2 - \frac{(R_2^2 - R_1^2 - a^2)}{a} d_1 + R_1^2 = 0$$

$$d_2 = \frac{R_2^2}{a + \frac{R_2^2 - R_1^2 - a^2 + \sqrt{(R^2 - R_1^2 - a^2)^2 - R_1^2}}{2a}} =$$

$$= \frac{2aR_2^2}{R_2^2 - R_1^2 + a^2 \pm \sqrt{R_2^2 - R_1^2 + a^2)^2 - 4R_1^2}} =$$

$$= \frac{2a R_2^2 (R_2^2 - R_1^2 + a^2 \mp \sqrt{(R_2^2 - R_1^2 - a^2)^2 - 4R_1^2}}{(R_2^2 - R_1^2 - a^2)^2 - \{(R_2^2 - R_1^2 - a^2)^2 - 4R_1^2\}} =$$

$$= \frac{R_2^2 - R_1^2 - a^2 \mp \sqrt{(R_2^2 - R_1^2 - a^2)^2 - 4R_1^2 a^2}}{2a} \Rightarrow$$

$d_1 + a < d_2 \Rightarrow$ берём „$+$"

$$d_1 d_2 = \frac{R_2^2 + R_1^2 - a^2 + \sqrt{(R_2^2 - R_1^2 - a^2)^2 - 4R_1^2 a^2}}{2a}$$

$$\varphi^{(1)} - \varphi^{(2)} = 2\varkappa \ln \frac{R_1 R_2}{d_1 d_2} = -2\varkappa \ln \frac{d_1 d_2}{R_1 R_2} =$$

$$= -2\varkappa \ln \left(\frac{2R_1R_2 \{ (R_2^2 + R_1^2 - a^2) + \sqrt{(R_2^2 - R_1^2 - a^2)^2 - 4R_1^2 a^2} \}}{(R_2^2 + R_1^2 - a^2)^2 - (R_2^2 - R_1^2 - a^2)^2 + 4R_1^2 a^2} \right) =$$

$$= 2\varkappa \ln \left(\frac{(R_2^2 + R_1^2 - a^2) + \sqrt{(R_2^2 - R_1^2 - a^2)^2 - 4R_1^2 a^2}}{2R_1 R_2} \right) =$$

$$= \left\{ (R_2^2 - R_1^2 - a^2)^2 - 4R_1^2 a^2 = (R_2^2 + R_1^2 - a^2)^2 - 4R_1^2 R_2^2 \right\} =$$

$$= 2\varkappa \ln \left(\frac{R_2^2 + R_1^2 - a^2}{2R_1 R_2} + \sqrt{\left(\frac{R_2^2 + R_1^2 - a^2}{2R_1 R_2}\right)^2 - 1} \right) =$$

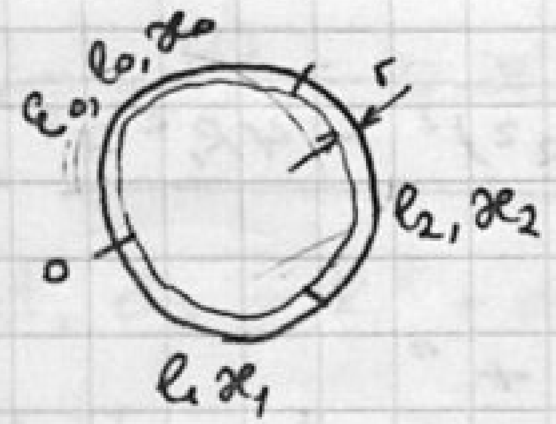
$$= 2\varkappa \,\mathrm{arcch} \left(\frac{R_2^2 + R_1^2 - a^2}{2R_1 R_2} \right)$$

$$C = \frac{1}{2\,\mathrm{arcch}\left(\frac{R_2^2 + R_1^2 - a^2}{2R_1 R_2} \right)}$$

$$c = \frac{}{2\,\mathrm{arcch}\left(\frac{R_2^2 + R_1^2 - a^2}{2R_1R_2}\right)}$$

$\boxed{\text{Доза } \#9}$ 229 232 235

#229



$\mathcal{E}_0 \neq \mathcal{E}_0(t)$

$E(2) - ?$

Запишем з-н Ома.

$$\vec{j} = \varkappa(\vec{E} + \vec{E}_{cm})$$

$$\frac{1}{\varkappa}\oint \vec{j}\,d\vec{l} = \int_1^2 \frac{j_1}{\varkappa_1}dl_1 + \int_2 \frac{j_2}{\varkappa_2}dl_2 + \int_0^1 \frac{j_0}{\varkappa_0}dl_0 =$$

$$- \frac{j_1}{\varkappa_1}l_1 + \frac{j_2}{\varkappa_2}l_2 + \frac{j_0}{\varkappa_0}l_0$$

из закона Ома имеем:

$$\oint \frac{\vec{j}}{\varkappa}d\vec{l} = \oint(\vec{E} + \vec{E}_{cm})d\vec{l} = \underbrace{\oint \bar{E}\,d\bar{l}}_{0} + \oint \bar{E}_{cm}\,d\bar{l} = E_{cm}l_0 = \mathcal{E}_0 \quad \text{ЭДС}$$

$$\operatorname{div}\vec{j}=0 \rightarrow j_{1n}=j_{2n}\big|_S$$

$$j_{1n}=j_{0n}\big|_S$$

$$j_{0n}=j_{2n}\big|_S$$

$$j=j_0=j_1=j_2 \rightarrow j\left(\frac{l_1}{\varkappa_1}+\frac{l_2}{\varkappa_2}+\frac{l_0}{\varkappa_0}\right)=\mathcal{E}_0$$

$$j=\mathcal{E}_0\frac{\varkappa_1\varkappa_2\varkappa_0}{(l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0+l_0\varkappa_1\varkappa_2)}$$

$$E_1=\frac{j_1}{\varkappa_1}=\frac{\mathcal{E}_0\varkappa_2\varkappa_0}{(l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0+l_0\varkappa_1\varkappa_2)}$$

$$E_2=\frac{j_2}{\varkappa_2}=\mathcal{E}_0\frac{\varkappa_1\varkappa_0}{l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0+l_0\varkappa_1\varkappa_2}$$

$$E_0=\frac{j_0}{\varkappa_0}-E_{cm}=\mathcal{E}_0\left\{\frac{\varkappa_1\varkappa_2}{l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0+l_0\varkappa_1\varkappa_2}-1\right\}=$$

$$=-\frac{\mathcal{E}_0}{l_0}\frac{l_1\varkappa_1\varkappa_0+l_2\varkappa_1\varkappa_2}{l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0+l_0\varkappa_1\varkappa_2}$$

$$E_{2n}-E_{1n}=4\pi\sigma=4\pi\frac{q_{21}}{\pi r^2};\quad q_{21}=\frac{r^2}{4}(E_2-E_1)=$$

$$=\frac{r^2}{4}\frac{\mathcal{E}_0\varkappa_0(\varkappa_2-\varkappa_1)}{l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0+l_0\varkappa_1\varkappa_2}$$

$$q_{10}=\frac{r^2}{4}(E_1-E_0)=\frac{r^2}{4}\frac{\mathcal{E}_0}{l_0}\frac{l_0\varkappa_2\varkappa_0+l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0}{l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0+l_0\varkappa_1\varkappa_2}$$

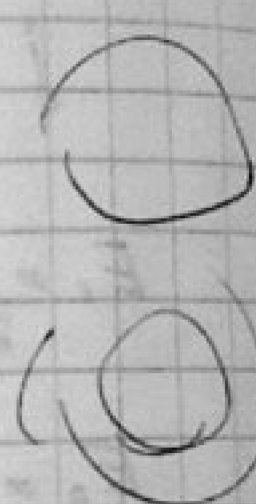
$$q_{20}=\frac{r^2}{4}(E_2-E_0)=\frac{r^2}{4}\frac{\mathcal{E}_0}{l_0}\frac{l_0\varkappa_1\varkappa_0+l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0}{l_1\varkappa_2\varkappa_0+l_2\varkappa_1\varkappa_0+l_0\varkappa_1\varkappa_2}$$

#232 а) $R = \int_a^в \frac{dr}{\varkappa S} = \int_a^в \frac{dr}{\varkappa 4\pi r^2} = \frac{1}{4\pi\varkappa} \frac{1}{r}\Big|_a^в$

б) $R = \int_a^c \frac{dr}{\varkappa_1 S} + \int_c^в \frac{dr}{\varkappa_2 S} = \frac{1}{4\pi\varkappa_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{4\pi\varkappa_2}\left(\frac{1}{c} - \frac{1}{в}\right)$

в) $R = \int_a^в \frac{dr}{\varkappa S} = \int_a^в \frac{dr}{\varkappa l 2\pi r} = \frac{1}{\varkappa l 2\pi} \ln r \Big|_a^в = \frac{1}{2\pi\varkappa l} \ln\frac{в}{a}$

#235 $C = \frac{q}{\Delta\varphi}$ $R = \int_0^d \frac{dr}{\varkappa S} = \frac{d}{\varkappa S}$; $R\varkappa = \frac{d}{S}$



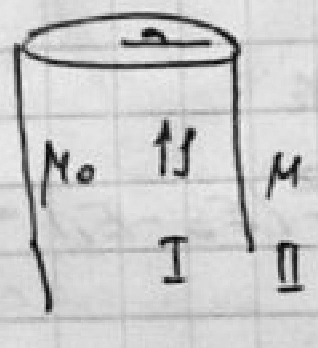
$C = \frac{\varepsilon q}{\Delta\varphi} = \frac{\varepsilon q}{4\pi\sigma r} = \frac{\varepsilon q}{4\pi\sigma d} = \frac{\varepsilon S}{4\pi d}$

$\int_0^d E dr$

$\Rightarrow C = \frac{\varepsilon}{4\pi R\varkappa}.$

Дома #10 242 243 244

#242.



$\vec{H}, \vec{B}$

1) $\oint \vec{H} d\vec{l} = \frac{4\pi J}{c}$; $\vec{B} = \mu\vec{H}$

I: $\int_0^{2\pi} H_\alpha r \, d\alpha = \frac{4\pi J(r)}{c}$

$H_\alpha r \cdot 2\pi = \frac{4\pi}{c} \frac{J}{\pi a^2} \pi r^2$

$H_\alpha = \frac{2Jr}{ca^2}$; $B_\alpha = \frac{\mu_0 2 J r}{ca^2}$

II $\oint \vec{H} d\vec{l} = \frac{4\pi}{c} J$

$\int_0^{2\pi} H_\alpha r \, d\alpha = \frac{4\pi}{c} J$; $H_\alpha = \frac{2J}{cr}$; $B_\alpha = \frac{\mu 2 J}{cr}$

2) $\Delta \vec{A} = -\frac{4\pi \mu \vec{j}}{c}$

$\Delta A_r = 0$

$\Delta A_\alpha = 0$

$\Delta A_z = -\frac{4\pi \mu j}{c}$

цилиндрическая симметрия

$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{d}{dr} A_z\right) = -\frac{4\pi \mu_0}{c} j$

$\left(r \frac{dA_z}{dr}\right) = -\frac{4\pi}{c}\mu_0 j \frac{r^2}{2} + C_1$

$A_{zI} = -\frac{\pi \mu_0}{c} j r^2 + \cancel{C_1 \ln r} + C_2$

$A_{zII} = -\cancel{\frac{\pi \mu_0}{c} j r^2} + C_3 \ln r + \cancel{C_4}$

$A_{1z} = A_{2z}\big|_{r=a}$ $\qquad A_{1z} = -\frac{\pi \mu_0 j}{c} r^2 + C_2$

$A_{2z} = C_3 \ln r$

$-\frac{\pi \mu_0 j}{c} a^2 + C_2 = -C_3 \ln r$

$\vec{B} = \operatorname{rot}\vec{A}$, дифференцируем $\frac{\partial A_{Iz}}{\partial r}$ и $\frac{\partial A_{zII}}{\partial r}$

$H_{1\perp} = H_{2\perp}\big|_{r=a} \rightarrow -\frac{2\pi \mu_0}{c} a j = \frac{C_3}{a}$; $C_3 = -\frac{2\pi \mu_0 a^2}{c} j$

$C_2 = -\frac{\pi \mu_0 j}{c} a^2 + \frac{2\pi \mu_0 a^2}{c} j \ln a$

$C_2 = -\frac{\pi a^2}{c}(2\mu \ln a - j \mu_0)$

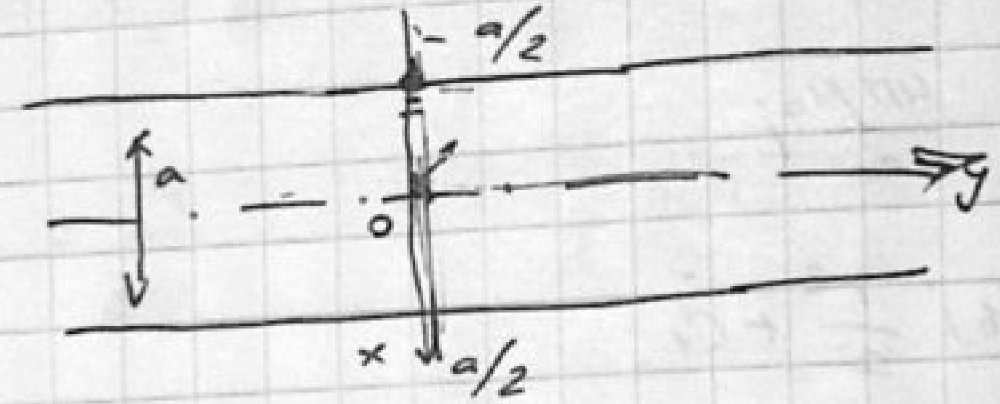
$A_{1z} = -\frac{\pi \mu_0}{c} j z^2 - \frac{\pi a^2}{c}(2\mu \ln a - j\mu_0)$

$A_{2z} = -\pi \frac{2\mu a^2}{c} j \ln r$

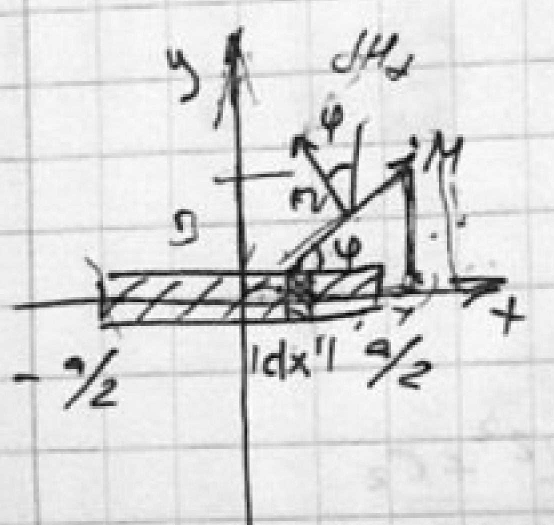
$$B_{12} = -\frac{\partial A_z}{\partial r} = \frac{2\pi \mu_0 r}{c}$$

$$B_{2\alpha} = \frac{2\pi \mu a^2 j}{cr}$$

244.



H - ?



$$dH_\alpha = \frac{2dI}{c r}$$

$$dI = \frac{I}{a} dx'$$

$$d\vec{H} = \frac{2dI}{cr} \vec{e_\alpha}$$

$$\begin{cases} e_y = \cos\varphi \\ e_x = \sin\varphi \end{cases}$$

$$\vec{e_\alpha} = (-\sin\varphi, \cos\varphi)$$

$$\sin\varphi = \frac{y}{r}; \quad \cos\varphi = \frac{x-x'}{r}$$

$$r = \sqrt{(x-x')^2 + y^2}$$

проектируем

$$\begin{cases} dH_x = \frac{2\,dI_x}{cr} = \frac{2I dx'}{cra}\left(-\frac{y}{r}\right) \\ dH_y = \frac{2I dx'}{cra}\left(\frac{x-x'}{r}\right) \end{cases}$$

$\downarrow$

243. $\vec{H}, \vec{B}$ - ?

$$\Delta \vec{A} = -\frac{4\pi\mu}{c}\vec{j}$$

$$\begin{cases} \Delta A_\alpha = 0 \\ \Delta A_r = 0 \\ \Delta A_z = -\frac{4\pi\mu}{c} j \end{cases}$$

аксиальная симметрия.

I область:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) = -\frac{4\pi\mu}{c} j_z = 0$$

$$A_z = C_1 \ln r + C_2$$

$\ln r \xrightarrow{r\to 0}$ расходится

$$A_z = C_2; \quad \vec{B} = 0$$

II область:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) = -\frac{4\pi\mu_0}{c} j$$

$$A_z = -\frac{2\pi\mu_0}{c} j \frac{r^2}{2} + C_3 \ln r + C_4$$

III область

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) = 0$$

$$A_z = C_5 \ln r + C_6$$

$$j = \frac{1}{S}\ ;\ S = \int_a^b r\,dr\,d\alpha = 2\pi \frac{r^2}{2}\Big|_a^b = \pi(b^2 - a^2)$$

$$j = \frac{1}{\pi(b^2-a^2)}$$

Гран. условие:

$$A_{1z} = A_{2z}\Big|_{r=a}$$

$$H_{1\alpha} = H_{2\alpha}\Big|_{r=a}$$

$$\frac{1}{\mu}\,\mathrm{rot}\,A_{1z} = \frac{1}{\mu_0}\,\mathrm{rot}\,A_{2z}\Big|_{r=a}$$

$$0 = -\frac{Ja}{c(b^2-a^2)} + C_3\frac{1}{a\mu_0} \Rightarrow C_3 = \frac{2Ja^2\mu_0}{c(b^2-a^2)}$$

$$A_{2z} = -\frac{\mu_0}{c}\frac{J}{(b^2-a^2)}r^2 + \frac{2Ja^2\mu_0}{c(b^2-a^2)}\ln r + C_4 =$$

$$= \frac{2Ja^2\mu_0}{c(b^2-a^2)}\left(\ln r - \frac{r^2}{2a^2}\right) + C_4$$

$$\frac{1}{\mu_0}\,\mathrm{rot}\,A_{2z} = \frac{1}{\mu}\,\mathrm{rot}\,A_{3z}\Big|_{r=b}$$

$$\frac{1}{\mu_0}\frac{2Ja^2\mu_0}{c(b^2-a^2)}\left(\frac{1}{b} - \frac{b}{a^2}\right) = \frac{1}{\mu}\frac{C_5}{b}$$

$$C_5 = -\frac{2J\mu}{c}$$

$$A_{1z} = A_{2z}\Big|_{r=a}$$

$$C_2 = \frac{2Ja^2\mu_0}{c(b^2-a^2)}\left(\ln a - \frac{1}{2}\right) + C_4$$

$$C_4 = C_2 - \frac{2Ja^2\mu_0}{c(b^2-a^2)}\ln a + \frac{Ja^2\mu_0}{c(b^2-a^2)}$$

$$A_{2z} = \frac{2ja^2\mu_0}{c(b^2-a^2)}\left[\ln\left(\frac{r}{a}\right) - \frac{r^2}{2a^2}\right] + \tilde{C_4}; \quad \tilde{C_4} = C_2$$

$$A_{2z} = A_3 \Big|_{r=b}$$

$$\frac{2ja^2\mu_0}{c(b^2-a^2)}\left(\ln\frac{b}{a} - \frac{b^2}{2a^2}\right) + C_4^{\,2} = -\frac{2j\mu\ln b}{c} + C_6$$

$$\to \frac{2ja^2\mu_0}{c(b^2-a^2)}\ln\frac{b}{a} - \frac{jb^2\mu_0}{c(b^2-a^2)} + C_2\frac{ja^2\mu_0}{c(b^2-a^2)} =$$

$$= -\frac{2j\mu}{c}\ln b + C_6$$

$$\underbrace{\frac{2ja^2\mu_0}{c(b^2-a^2)}\ln\frac{b}{a} + C_2 - \frac{j\mu_0}{c}}_{\tilde{C_6}} = -\frac{2j\mu\ln b}{c} + c$$

$$C_6 = \frac{2j\mu}{c}\ln b + \tilde{C_6}$$

$$A_{3z} = \frac{2j\mu}{c}\ln\frac{b}{r} + \tilde{C_6}$$

$\Rightarrow$ Ответ: $A_1 = C_2$, $\vec{B_1} = 0$

$$A_{2z} = \frac{2\mu_0 ja^2}{c(b^2-a^2)}\left(\ln\frac{r}{a} - \frac{r^2}{2a^2}\right) + \tilde{C_4}$$

$$B_{2\alpha} = \frac{2\mu_0 j}{c(b^2-a^2)}\left(r - \frac{a^2}{r}\right)$$

$$A_{3z} = \frac{2j\mu}{c}\ln\frac{b}{r} + \tilde{C_6}$$

$$B_{3\alpha} = \frac{2j\mu}{cr}$$

244 продолж.

$$dH_x = \int_{-a/2}^{a/2} \left(-\frac{2J}{ca} y\right) \frac{dx'}{r^2}$$

$$H_y = \frac{2J}{car} \int_{-a/2}^{a/2} \frac{(x-x')}{r} dx'$$

$$H_x = \frac{2J_y}{ca} \int_{-a/2}^{a/2} \frac{d(x-x')}{(x-x')^2+y^2} = \frac{1}{y} \operatorname{arctg} \frac{(x-x')}{y} \left(\frac{2J y}{ca}\right) \Big|_{-a/2}^{a/2} =$$

$$= \frac{2J}{c} \operatorname{arctg} \left(\frac{x-x'}{y}\right) \Big|_{-a/2}^{a/2} =$$

$$= \frac{2J}{c} \left[\operatorname{arctg} \left(\frac{x-a/2}{y}\right) - \operatorname{arctg} \left(\frac{x+a/2}{y}\right)\right]$$

$$H_y = \int_{-a/2}^{a/2} \frac{2J dx'}{cra} \frac{(x-x')}{r} = \frac{2J}{c\,a} \int_{-a/2}^{a/2} \frac{dx'(x-x')}{(x-x')^2+y^2} =$$

$$= -\frac{2J}{ca} \int_{-a/2}^{a/2} \frac{1}{2} \frac{d\left[(x-x')^2+y^2\right]}{\left[(x-x')^2+y^2\right]} =$$

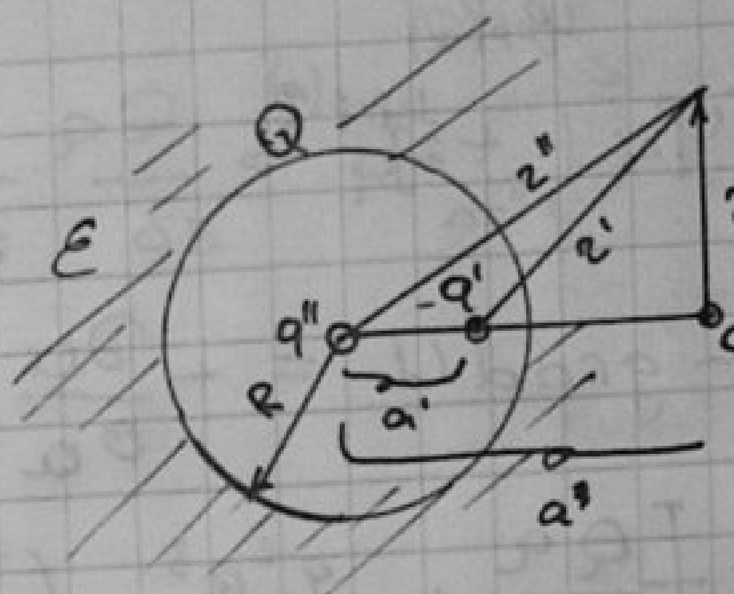
$$= -\frac{J}{ca} \ln\left[(x-x')^2+y^2\right] \Big|_{-a/2}^{a/2} = -\frac{J}{ca} \ln \frac{(x-a/2)^2+y^2}{(x+a/2)^2+y^2}$$

Доза #6

#162.

Q, a, R, q, ε

$U, F - ?$



$$U = \frac{1}{2} \varphi_{сф}\Big|_{r=a} \cdot q$$

Если $\varphi_{сф} \sim q \Rightarrow U = \frac{1}{2}\varphi_{сф} q$

$\varphi_{сф} \neq \varphi_{сф}(q) \Rightarrow U = \varphi_{сф} q.$

$$\begin{cases} \dfrac{q^2}{q'^2} = \dfrac{a}{a'} \\ R^2 = aa' \end{cases} \Rightarrow a' = \frac{R^2}{a} ; \quad q' = q\frac{R}{a}$$

$$q'' = Q + q\frac{R}{a} ; \quad \varphi_{сф} = \frac{q''}{\varepsilon r''} - \frac{q'}{\varepsilon r'} =$$

$$= \frac{Q}{\varepsilon r''} + \frac{q}{\varepsilon r''}\frac{R}{a} - \frac{q}{\varepsilon r'}\frac{R}{a} = \left\{ \begin{array}{l} r' = \sqrt{a'^2 + r''^2 - 2a'r''\cos\Theta} \\ r'' = a \end{array} \right\} =$$

$$= \frac{Q}{\varepsilon a} + \frac{qR}{\varepsilon a^2} - \frac{qR}{\varepsilon a \sqrt{a'^2 + r''^2 - 2a'r''\cos\Theta}} =$$

$$= \frac{Q}{\varepsilon a} + \frac{qR}{\varepsilon a^2} - \frac{qR}{\varepsilon a\left(\frac{a'^2 - a^2}{a}\right)} = \frac{Q}{\varepsilon a} + \frac{qR}{\varepsilon a^2} - \frac{qR}{\varepsilon (R^2 - a^2)} =$$

$$= \frac{Q}{\varepsilon a} - \frac{qR^3}{\varepsilon(a^2-R^2)a^2}$$

$\varphi_{сф}(Q)$

$$U = \left[\varphi_{сф}\Big|_{r''=a} + \frac{1}{2}\varphi_{сф}(q)\right]q = \frac{Qq}{\varepsilon a} - \frac{q^2R^3}{2\varepsilon(a^2-R^2)a^2}$$

$$\vec{F} = -\operatorname{grad} U = -\frac{\partial U}{\partial a};$$

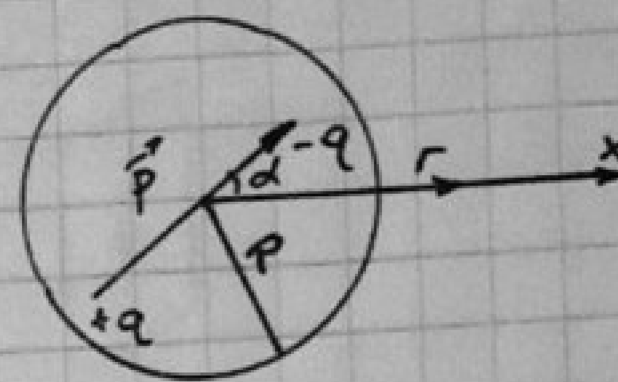
$$\Rightarrow \vec{F} = -\left[-\frac{Qq}{\varepsilon a^2} + \frac{q^2R^3}{2\varepsilon}\left(\frac{4a^3-2aR^2}{a^4(a^2-R^2)^2}\right)\right] =$$

$$= \frac{Qq}{\varepsilon a^2} - \frac{q^2R^3(2a^2-2R^2)}{\varepsilon a^3(a^2-R^2)}!$$

165.

$R, \vec{p}.$

$\sigma, E' - ?$



$E|_{r=R} = 4\pi\sigma;$

$$\Rightarrow \sigma = \frac{E}{4\pi} = \frac{-\operatorname{grad}\varphi}{4\pi}$$

$$\Delta\varphi = 0.$$

$$\varphi = \varphi_{дип} + \varphi_{инд}$$

$$\varphi_{дип} = \frac{\vec{p}\vec{r}}{r^3};$$

$$\Delta\varphi_{инд} = 0 \Rightarrow \varphi_{инд} = \vec{A}\vec{R}: \qquad E_{инд} = -\operatorname{grad}\varphi_{инд}.$$

$$E_{инд} = \frac{p}{R^3}$$

$$\varphi|_{r=R} = \frac{\bar{p}\bar{r}}{\bar{R}^3} + \vec{A}\bar{R} = 0 \Rightarrow \vec{A} = -\frac{\vec{p}}{\vec{R}^3}$$

$$\Rightarrow \varphi = \frac{\vec{p}\vec{r}}{r^3} - \frac{\bar{p}\vec{r}}{\bar{R}^3}, \qquad (\bar{p}\bar{r}) = pr\cos\alpha$$

$$\Rightarrow \varphi = \frac{p\cancel{r}\cos\alpha}{r^{\cancel{3}2}} - \frac{pr\cos\alpha}{R^3})$$

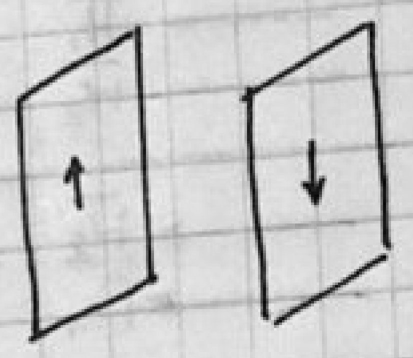
$$\sigma = -\frac{\partial}{\partial r}\frac{1}{4\pi}\left[\frac{p\cos\alpha}{r^2} - \frac{pr\cos\alpha}{R^3}\right] = \{r = R\} = \frac{3p\cos\alpha}{4\pi R^3};$$

$$\bar{E}' = -\operatorname{grad}\varphi_{инд} = -\frac{\partial}{\partial r}\left(-\frac{\bar{p}r}{R^3}\right) = \frac{\vec{p}}{R^3}$$

Доза # 11

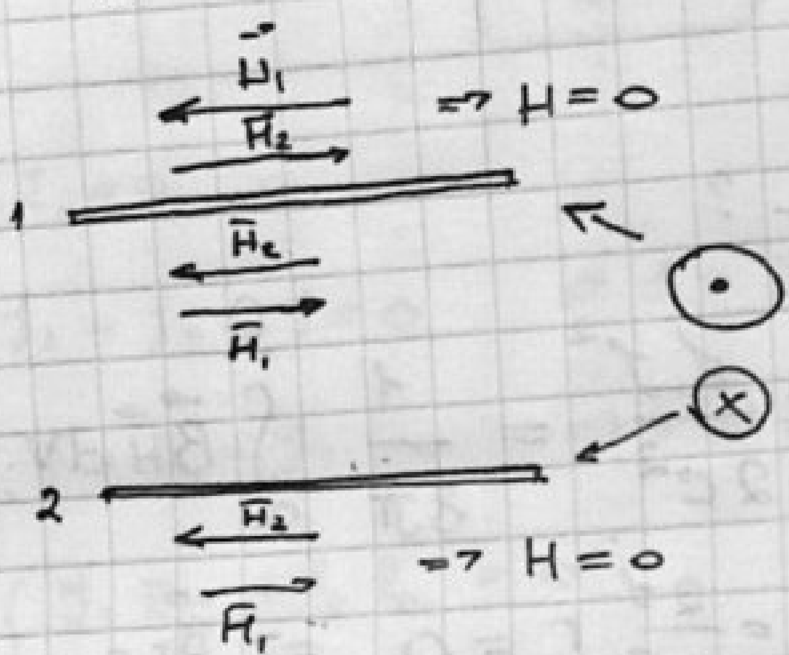
#247 $i = const$ ← поверхн. плотности.

$\vec{H}$ - ? а) ↑↓ б) ↑↑



$i = \frac{J}{a}$

а)



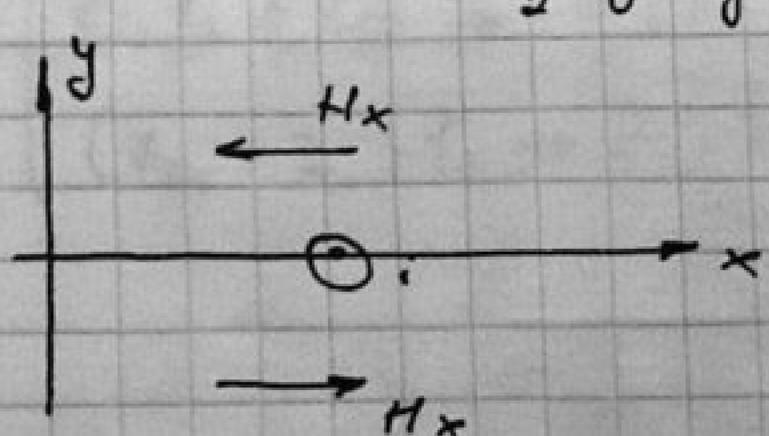
$$\underbrace{H = H_1 + H_2}$$

из #244

$$\cancel{H_y = \frac{2J}{ac}\ln\frac{y-\frac{a}{2}}{y+a/2}}$$

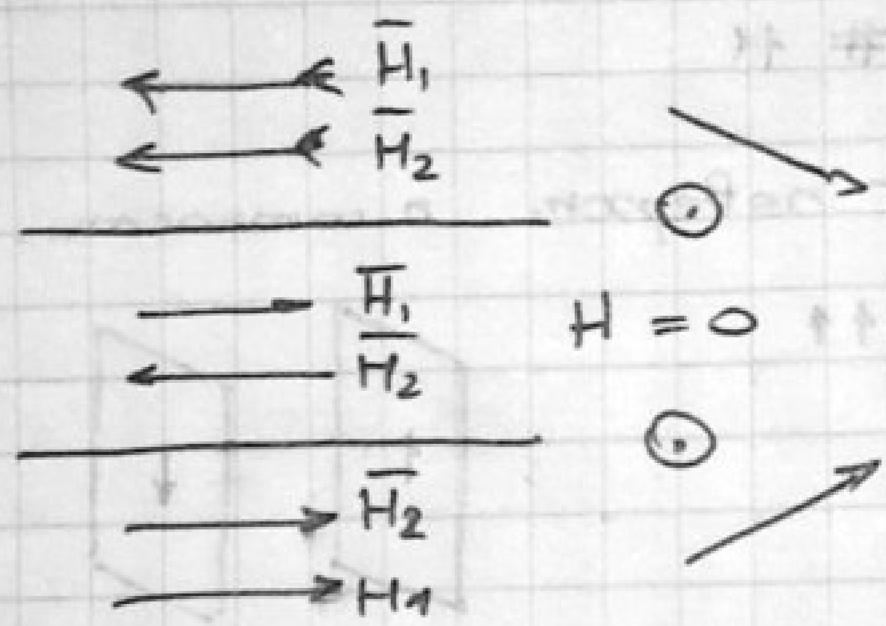
$$H_y = -\frac{J}{ca}\ln\frac{(x-a/2)^2+y^2}{(x+a/2)^2+y^2} = -\frac{2i}{c}\ln\frac{a^2/4+y^2}{a^2/4+y^2} \Rightarrow 0$$

$$H_x = -\frac{2i}{c}\left(\underbrace{2\,\mathrm{arctg}\,\frac{a}{2y}}_{\frac{\pi}{2}\mathrm{sgn}\,y}\right) = -\frac{2\pi i}{c}\,\mathrm{sgn}\,y$$



$\Rightarrow H = \frac{4\pi i}{c}$

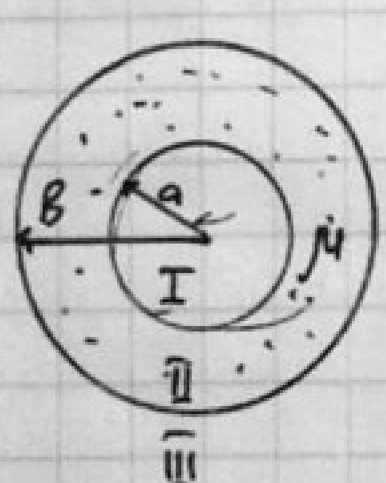
б)



$H = 0$ $\qquad H = \frac{4\pi i}{c}$

#263

$L - ?$



$$W = \frac{LI^2}{2c^2} = \frac{1}{8\pi}\int_a^b \vec{B}\vec{H}\,dV;$$

из 241

I : $H_\alpha = \frac{2I}{ca^2} r = 0 \Rightarrow \vec{B}_I = 0$

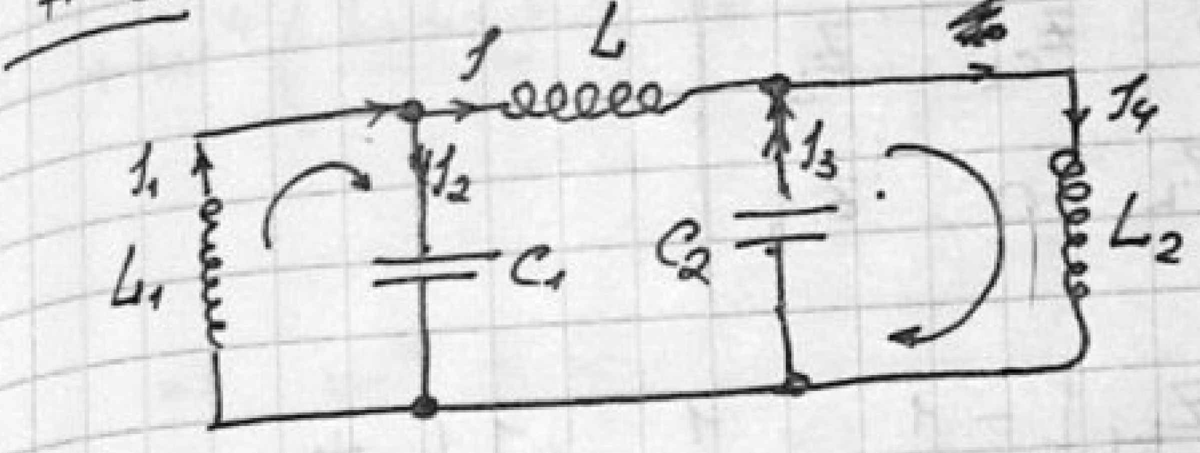
II. $H_\alpha = \frac{2I}{c}\frac{1}{r}$; $\vec{B}_{II} = \frac{\mu I}{c^2}$

$$\Rightarrow W = \frac{\mu}{8\pi}\int_a^b \frac{I^2}{c^2 r^2}\, r\, 2\pi\, dr = \frac{I^2\mu}{c^2}\ln\frac{b}{a}$$

$$\Rightarrow L = 2\mu \ln\frac{b}{a}$$

Д/оза # 12

355



1) $\Sigma I_i = 0$

$$\begin{cases} I_1 - I_2 - I = 0 & \Rightarrow I = I_1 - I_2 \\ I + I_3 - I_4 = 0 & \Rightarrow I = I_4 - I_3 \end{cases}$$

2) $$\begin{cases} I_1 Z_{L_1} + I_2 Z_{C_1} = 0 \\ I_3 Z_{C_2} + I_4 Z_{L_2} = 0 \\ I_1 Z_{L_1} + I Z_L + I_4 Z_{L_2} \end{cases}$$

$$Z_{L_1} = -\frac{i\omega L_1}{c^2} \qquad Z_{L_2} = -\frac{i\omega L_2}{c^2} \qquad Z_L = -\frac{i\omega L}{c^2}$$

$$Z_{C_1} = \frac{i}{\omega C_1} \qquad Z_{C_2} = \frac{i}{\omega C_2}$$

$$\Rightarrow \begin{cases} I_1 - I_2 + I_3 - I_4 = 0 \\ I_1 Z_{L_1} + Z_{C_1} I_2 = 0 \\ Z_{C_2} I_3 + Z_{C_2} I_4 = 0 \\ (Z_{L_1} + Z_L) I_1 - Z I_2 + Z_{L_2} I_4 = 0 \end{cases}$$

$$\det = \left|\begin{matrix} 1 & -1 & 1 & -1 \\ Z_{L_1} & Z_{C_1} & 0 & 0 \\ 0 & 0 & Z_{C_2} & Z_{L_2} \\ Z_{L_1}+Z_L & -Z_L & 0 & Z_{L_2} \end{matrix}\right| =$$

$$= - Z_{L_1}\left|\begin{matrix} -1 & 1 & 1 \\ 0 & Z_{C_2} & Z_{L_2} \\ -Z_L & 0 & Z_{L_2} \end{matrix}\right| +$$

$$+ Z_{C_1}\left|\begin{matrix} 1 & 1 & -1 \\ 0 & Z_{C_2} & Z_{L_2} \\ Z_{L_1}+Z_L & 0 & Z_{L_2} \end{matrix}\right| =$$

$$= +Z_{L_1} Z_{C_2} L_{L_2} + Z_{L_1} Z_{L_2} Z_L - Z_{L_1} Z_L Z_{C_2} +$$

$$+ Z_{C_1} Z_{C_2} Z_{L_2} + Z_{C_1} Z_{L_2}(Z_{L_1}+Z_L) - Z_{C_1} Z_{C_2}(Z_{L_1}+Z_L) = 0$$

$$= +\frac{i\omega L_1}{c^2}\,\frac{i}{\cancel{\omega} C_2}\,\frac{i\cancel{\omega} L_2}{c^2} + -\frac{i\omega L_2}{c^2}\,\frac{i\omega L_2}{c^2}\,\frac{i\omega L}{c^2} -$$

$$- \frac{i\omega L_1}{c^2}\,\frac{i\cancel{\omega} L}{c^2}\,\frac{i}{C_2\cancel{\omega}} + \frac{i}{\omega C_1}\,\frac{i}{\cancel{\omega} C_2}\,\frac{i\cancel{\omega} L_2}{c^2} + \frac{i}{\cancel{\omega} C_1}\,\frac{i\cancel{\omega} L_2}{c^2}\left(\frac{i\omega L_1 + i\omega L}{c^2}\right) +$$

$$+ \frac{i}{\omega C_1}\,\frac{i}{\cancel{\omega} C_2}\,\frac{i\cancel{\omega}}{c^2}(L_1+L) =$$

$$= -\frac{\cancel{i}\omega L_1 L_2}{c^{\cancel{4}2} C_2} + \frac{\cancel{i}\omega^3 L_1 L_2 L}{c^{\cancel{6}4}} + \frac{\cancel{i}\omega L_1 L}{c^{\cancel{4}2} C_2} + \frac{\cancel{i} L_2}{\omega C_1 C_2 \cancel{c^2}} -$$

$$- \frac{\cancel{i} L_2 (L_1+L)\omega}{C_1 c^{\cancel{4}2}} - \frac{\cancel{i}(L_1+L)}{\omega C_1 C_2 \cancel{c^2}} =$$

$$\omega^2\left(-\cancel{\frac{L_1 L_2}{c^2 c_2}}+\cancel{\frac{L_1 L_2}{c^2 c_2}}-\frac{L_2(L_1+L)}{c_1 c^2}\right)+$$

$$+\omega^4\left(\frac{L_1 L_2 L}{c^4}\right)+\left[\frac{L_2}{c_1 c_2}-\frac{(L_1+L)}{c_1 c_2}\right]=0$$

$$\Rightarrow \quad \omega^2 = x.$$

$$x^2\left[\frac{L_1 L_2 L}{c^4}\right]-x\left[\frac{L_2(L_1+L)}{c_1 c^2}\right]+\frac{L_2-L_1-L}{c_1 c_2}=0$$

$$x_{1,2}=\frac{\dfrac{L_2(L_1+L)}{c_1 c^2}\pm\sqrt{\dfrac{L_2^2(L_1+L)^2}{c_1^2 c^4}-4\dfrac{L_1 L_2 L}{c^4}\dfrac{(L_2-L_1-L)}{c_1 c_2}}}{2\dfrac{L_1 L_2 L}{c^4}}=$$

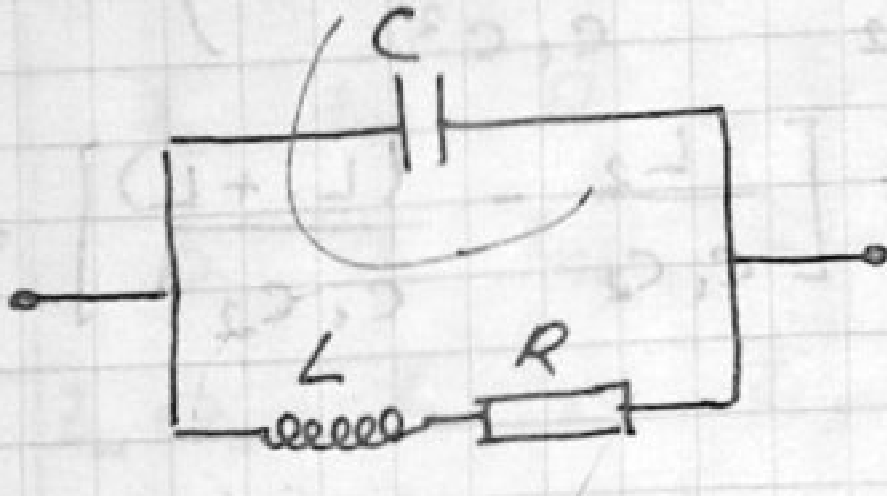
$$=\frac{L_2(L_1+L)}{c_1 c^2}\pm\frac{1}{c^2}$$

$$\omega_{1,2}=\frac{c^2}{2}\left(\frac{1}{c_1}\left(\frac{1}{L_1}+\frac{1}{L}\right)+\frac{1}{c_2}\left[\frac{1}{L_2}+\frac{1}{L}\right]\right)\pm$$

$$\pm\frac{c^2}{2}\cancel{4}\left\{\left[\frac{1}{c_1}\left(\frac{1}{L_1}+\frac{1}{L}\right)-\frac{1}{c_2}\left(\frac{1}{L_2}+\frac{1}{L}\right)\right]^2+\right.$$

$$\left.+\frac{4}{L^2 c_1 c_2}\right\}^{1/2}$$

359



Z – ?

$$Z_C = \frac{i}{\omega C}$$

$$Z_L = -i\frac{\omega L}{c^2}$$

$$Z_R = R$$

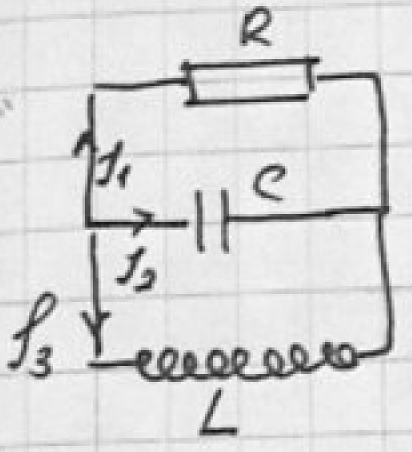
$$\frac{1}{Z} = \frac{1}{Z_c} + \frac{1}{Z_L + Z_R} = \frac{Z_L + Z_R + Z_C}{Z_c(Z_L + Z_R)} \Rightarrow$$

$$\Rightarrow Z = \frac{\frac{i}{\omega C}\left(-\frac{i\omega L}{c^2} + R\right)}{-\frac{i\omega L}{c^2} + R + \frac{i}{\omega C}} = \cancel{\frac{\frac{L}{cc^2} + \frac{iR}{\omega C}}{\quad}}$$

$$= \frac{\left[\frac{R}{\omega c} - \frac{L}{cc^2}\right]\frac{1}{i}}{-\left[R + i\left(\frac{1}{\omega C} - \frac{\omega L}{c^2}\right)\right]} = \frac{R - \frac{iL\omega}{c^2}}{-\left[iR\omega C - \left(1 - \underbrace{\frac{\omega^2 LC}{c^2}}_{\omega_1^2}\right)\right]} =$$

$$= \frac{R - \frac{i\omega L}{c^2}}{1 - iR\omega C - \frac{\omega^2}{\omega_1^2}}$$

#59.



1й закон Кирхгофа:

$$\left\{ I_1 + I_2 + I_3 = 0 \right.$$

2й закон Кирхгофа: $\qquad Z_R = R.$

$$\begin{cases} I_1 Z_R - Z_C I_2 = 0 \\ I_1 Z_R - I_3 Z_L = 0 \end{cases} \qquad Z_C = \frac{i}{\omega C} \qquad Z_L = -\frac{i\omega L}{c^2}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ R & -Z_C & 0 \\ R & 0 & -Z_L \end{vmatrix} = Z_C Z_L + Z_C R + Z_L R = 0$$

$$\Rightarrow -\frac{i}{\cancel{\omega} C} \frac{i\omega L}{c^2} + \frac{i}{\omega C} R - \frac{i\omega L}{c^2} R = 0 \quad | \times \omega$$

$$\Rightarrow \quad \omega^2 \frac{L}{c^2} R + \frac{iL}{Cc^2} \omega - \frac{R}{C} = 0$$

~~[illegible]~~ $\quad \omega^2 + \frac{i}{RC}\omega - \frac{c^2}{LC} = 0$

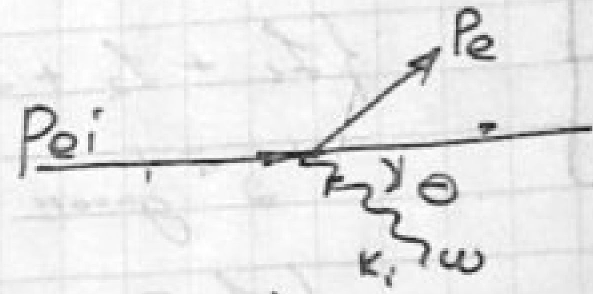
$$\frac{c^2}{LC} = \omega_0^2$$

$$\Rightarrow \omega_{1,2} = -i\frac{1}{2RC} \pm \sqrt{\omega_0^2 - \frac{1}{4R^2C^2}}$$

ницы объема.

Доза #13.

#676 $v > \dfrac{c}{n(\omega)}$; v, ω.



фотон: $k_i = \left(\dfrac{\hbar\omega}{c};\ \dfrac{\hbar\omega}{c} n(\omega)\vec{e}_k\right)$ $\qquad k_i^2 = \left(\dfrac{\hbar\omega}{2}\right)^2\left[1-n^2(\omega)\right]$

электрон $p_i = \left(\dfrac{\varepsilon}{c};\ \vec{p}\right)$; $\varepsilon = \dfrac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$; $\vec{p} = \dfrac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$; $p_i^2 = m^2c^2$

З.С.И $\quad p_{ei} + k_i = p_i \quad$ ^2.

$$\underline{m^2c^2} + 2\left(\varepsilon\frac{\hbar\omega}{c^2} - \frac{\hbar\omega}{c} n(\omega)\underbrace{\vec{e}_k\vec{p}}_{=p\cos\theta}\right) + \left(\frac{\hbar\omega}{c}\right)^2\left(1-n^2(\omega)\right) = \underline{m^2c^2}$$

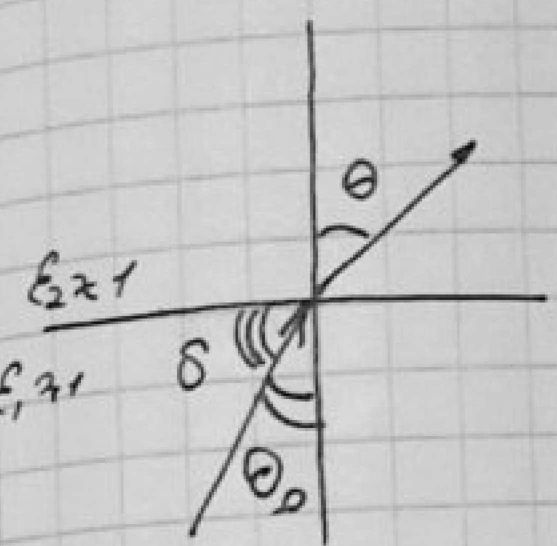
$$\cos\theta = \frac{\varepsilon}{pc\,n(\omega)} + \frac{\hbar\omega}{2cp\,n(\omega)}\left[1-n^2(\omega)\right]$$

$\dfrac{\varepsilon}{pc} = \dfrac{c}{v} = \beta \leq 1$ $\qquad$ можно пренебречь квантовой поправкой

$$\cos\theta \simeq \frac{c}{v\,n(\omega)} \leq 1$$

$$\Rightarrow \frac{c}{n(\omega)} \leq v \quad \text{ч.т.д.}$$

64.



$$R_\perp = \left| \frac{\sqrt{\varepsilon_1}\cos\theta_0 - \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_0}}{\sqrt{\varepsilon_1}\cos\theta_0 + \sqrt{\varepsilon_2 - \varepsilon_1 \sin^2\theta_0}} \right|^2$$

$$R_{\parallel} = \left| \frac{\varepsilon_2\cos\theta_0 - \sqrt{\varepsilon_1(\varepsilon_2 - \varepsilon_1 \sin^2\theta_0)}}{\varepsilon_2\cos\theta_0 + \sqrt{\varepsilon_1(\varepsilon_2 - \varepsilon_1 \sin^2\theta)}} \right|^2$$

$$\delta = \frac{\pi}{2} - \theta_0 \; ; \; \theta_0 = \frac{\pi}{2} - \delta .$$

$$\cos\left(\frac{\pi}{2} - \delta\right) = \cancel{\cos\frac{\pi}{2}\cos\delta} + \sin\frac{\pi}{2}\sin\delta \approx \delta .$$

$$\sin^2\left(\frac{\pi}{2} - \delta\right) = 1 - \cos^2\left(\frac{\pi}{2} - \delta\right) \approx \underline{1 - \delta^2}$$

$$R_\perp = \left| \frac{\delta - \sqrt{\varepsilon_2 - (1-\delta^2)}}{\delta + \sqrt{\varepsilon_2 - (1-\delta^2)}} \right|^2$$

$$R_{\parallel} = \left| \frac{\varepsilon_2\,\delta - \sqrt{\varepsilon_2 - (1-\delta^2)}}{\varepsilon_2\,\delta + \sqrt{\varepsilon_2 - (1-\delta^2)}} \right|^2 .$$

При $\varepsilon_2 \delta \approx \delta$

$$\cancel{R_\perp \approx R_{\parallel} \approx \left| \frac{\delta - \sqrt{\varepsilon_2 - (1+\delta)^2}}{\delta^2 - \varepsilon_2 + 1 - \delta^2} \right|^2 = \frac{\left(\delta - \sqrt{\varepsilon_2 - (1+\delta)}\right)^4}{(\varepsilon_2 - 1)^2}}$$

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