Доза № 1

№ 39

а) $\operatorname{grad}(\varphi\psi) = \nabla(\varphi\psi) = \nabla(\overset{\downarrow}{\varphi}\psi) + \nabla(\varphi\overset{\downarrow}{\psi}) = \psi \operatorname{grad}\varphi +$

$+ \varphi \operatorname{grad}\psi$

б) $\operatorname{div}(\varphi\vec{A}) = \nabla(\varphi\vec{A}) = \nabla(\overset{\downarrow}{\varphi}\vec{A}) + \nabla(\varphi\overset{\downarrow}{\vec{A}}) = \vec{A}\cdot\operatorname{grad}\varphi + \varphi\cdot\operatorname{div}\vec{A}$

в) $\operatorname{rot}(\varphi\vec{A}) = \nabla\times(\varphi\vec{A}) = (\nabla\times\varphi\vec{A})_i = \varepsilon_{ikm}\cdot\dfrac{\partial}{\partial x_k}(\varphi\vec{A})_m =$

$= \varepsilon_{ikm}\left(\varphi\dfrac{\partial}{\partial x_k}\cdot a_m\right) + a_m\cdot\dfrac{\partial}{\partial x_k}\varphi) = \varphi \operatorname{rot}\vec{a} + \varepsilon_{ikm}\cdot a_m(\operatorname{grad}\varphi)_k =$

$= \varphi\operatorname{rot}\vec{a} - \varepsilon_{imk}\cdot a_m(\operatorname{grad}\varphi)_k = \varphi\operatorname{rot}\vec{a} - \vec{a}\times\operatorname{grad}\varphi$

г) $\operatorname{div}(\vec{A}\times\vec{B}) = \nabla(\vec{A}\times\vec{B}) = \nabla(\overset{\downarrow}{\vec{A}}\times\vec{B}) + \nabla(\vec{A}\times\overset{\downarrow}{\vec{B}}) =$

$= \vec{B}(\nabla\times\vec{A}) - \vec{A}(\nabla\times\vec{B}) = \vec{B}\cdot\operatorname{rot}\vec{A} - \vec{A}\cdot\operatorname{rot}\vec{B}$

д) $\operatorname{rot}(\vec{A}\times\vec{B}) = \nabla\times(\overset{\downarrow}{\vec{A}}\times\vec{B}) + \nabla\times(\vec{A}\times\overset{\downarrow}{\vec{B}}) = \vec{A}(\nabla\vec{B}) - \vec{B}(\nabla\vec{A}) +$

$+ \overset{\downarrow}{\vec{A}}(\nabla\vec{B}) - \overset{\downarrow}{\vec{B}}(\nabla\vec{A}) = \vec{A}\cdot\operatorname{div}\vec{B} - \vec{B}\operatorname{div}A + (\vec{B}\nabla)\vec{A} - (\vec{A}\nabla)\vec{B}$

е) $\operatorname{grad}(\vec{A}\cdot\vec{B}) = \nabla(\overset{\downarrow}{\vec{A}}\cdot\vec{B}) + \nabla(\vec{A}\cdot\overset{\downarrow}{\vec{B}}) = \vec{B}\times\operatorname{rot}\vec{A} +$

$\left|\begin{array}{l} \vec{A}\times(\nabla\times\overset{\downarrow}{\vec{B}}) = \nabla(\vec{A}\cdot\overset{\downarrow}{\vec{B}}) - (\vec{A}\nabla)\overset{\downarrow}{\vec{B}} \\ \vec{B}\times(\nabla\times\overset{\downarrow}{\vec{A}}) = \nabla(\overset{\downarrow}{\vec{A}}\cdot\vec{B}) - (\vec{B}\nabla)\overset{\downarrow}{\vec{A}} \end{array}\right|$ $\begin{array}{l} + (\vec{B}\nabla)\vec{A} + \\ + \vec{A}\times\operatorname{rot}\vec{B} + (\vec{A}\nabla)\vec{B} \end{array}$

$$\operatorname{grad} y(r) = (\nabla y(r))_i = \frac{\partial}{\partial x_i} y(r) = \frac{\partial y(r)}{\partial r} \cdot \frac{\partial r}{\partial x_i} =$$

$$= y' \cdot \frac{x_i}{r} = y'(r) \cdot \frac{\vec{r}}{r}$$

$$\operatorname{div} y(r)\vec{r} = \delta_{ik} \cdot \frac{\partial}{\partial x_i} (y(r) \cdot \vec{r})_k = \delta_{ik} \cdot \frac{\partial}{\partial x_i} y(r) \cdot x_k =$$

$$= \frac{\partial}{\partial x_i} y(r) \cdot x_i = \delta_{ii} y(r) + x_i \cdot \frac{\partial y(r)}{\partial x_i} = 3y(r) + \vec{r} \cdot \frac{\partial y(r)}{\partial r} \frac{\vec{r}}{r} =$$

$$= 3y(r) + y'(r) \cdot \frac{r^2}{r} = 3y(r) + y'(r) \cdot r$$

$$\operatorname{rot} y(r) \cdot \vec{r} = (\nabla \times y(r) \vec{r})_i = \varepsilon_{ijk} \cdot \frac{\partial}{\partial x_j} (y(r) \cdot \vec{r})_k = \varepsilon_{ijk} \frac{\partial}{\partial x_j} (y(r) x_k) =$$

$$= \varepsilon_{ijk} x_k \frac{\partial y}{\partial r} \cdot \overset{\frac{x_j}{r}}{\overset{\shortparallel}{\frac{\partial r}{\partial x_j}}} + \varepsilon_{ijk} y(r) \cdot \frac{\partial}{\partial x_j} x_k = \varepsilon_{ijk} \cdot x_k \cdot y'(r) \cdot \frac{x_j}{r} +$$

$$+ \varepsilon_{ijk} \cdot \delta_{jk} \cdot y(r) = \varepsilon_{ijk} \cdot x_j x_k \frac{y'(r)}{r} + \overset{0}{\overset{\shortparallel}{\varepsilon_{ikk}}} \cdot y(r) = \frac{y'(r)}{r} \overset{0}{\overset{\shortparallel}{(\vec{r} \times \vec{r})}} = 0$$

$$(\vec{1}\nabla) y(r) \cdot \vec{r} = (\vec{1}\nabla) \overset{\downarrow}{y}(r) \cdot \vec{r} + (\vec{1}\nabla) y(r) \cdot \overset{\downarrow}{\vec{r}} = \vec{r}\,(1_i \nabla_i)\, y(r) +$$

$$+ y(r)(1_i \nabla_i) x_j = \vec{r}\left(1_i \cdot \frac{\partial y}{\partial r} \cdot \frac{x_i}{r}\right) + y(r)(1_i \delta_{ij}) =$$

$$= \vec{r}\left(1_i x_i \frac{y'(r)}{r}\right) + y(r) \cdot \vec{1} = \vec{r}\,(\vec{1} \cdot \vec{r}) \cdot \frac{y'(r)}{r} + \vec{1} \cdot y(r)$$

$\nabla(\vec a\vec r) \to \frac{\partial}{\partial x_i} a_k x_k = a_i \to \vec a$

43. $\nabla \times \{(\vec a\vec r)\vec r\} = \nabla(\vec a\vec r) \times \vec r + (\vec a\vec r)\,\mathrm{rot}\,\vec r$, $\quad \nabla(\vec a\vec r)\times\vec r = \vec a \times \vec r$, $\quad \mathrm{rot}\,\vec r = 0$

$$\mathrm{rot}((\vec a\cdot\vec r)\cdot\vec r) = \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j}((\vec a\cdot\vec r)\cdot\vec r)_k = \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j}(\vec a\cdot\vec r)\,x_k =$$

$$= \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j}\delta_{mn}\cdot a_m\cdot x_n\cdot x_k = \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j} a_n x_n x_k =$$

$$= \varepsilon_{ijk}\left(a_n\cdot\frac{\partial x_n}{\partial x_j}x_k + a_n x_n\cdot\frac{\partial x_k}{\partial x_j}\right) = \varepsilon_{ijk}\,a_n(\delta_{jn}\cdot x_k + \delta_{jk}\cdot x_n) =$$

$$= \varepsilon_{ijk}\,a_j x_k + \varepsilon_{ijk}\cdot\delta_{jk}\cdot a_n x_n = \vec a\times\vec r$$

$$\mathrm{rot}(\vec a\cdot\vec r)\,\vec b = \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j}((\vec a\cdot\vec r)\cdot\vec b)_k = \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j}\delta_{ml}\,a_m x_l\,b_k =$$

$$= \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j} a_l\cdot x_l\,b_k = \varepsilon_{ijk}\cdot a_l\cdot b_k\cdot\delta_{jl} = \varepsilon_{ijk}\cdot a_j\,b_k = \vec a\times\vec b$$

$$\mathrm{div}(\vec a\cdot\vec r)\,\vec b = \delta_{ik}\cdot\frac{\partial}{\partial x_i} a_m\cdot x_m\cdot b_k = b_i\,a_m\,\delta_{im} = b_i\,a_i = \vec a\cdot\vec b$$

$$\mathrm{rot}(\vec a\times\vec r) = \varepsilon_{ijk}\,\frac{\partial}{\partial x_j}(\vec a\times\vec r)_k = \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j}\varepsilon_{klm}\cdot a_l\cdot x_m =$$

$$= \varepsilon_{ijk}\cdot\varepsilon_{klm}\,a_l\cdot\frac{\partial}{\partial x_j}x_m = a_l\cdot\delta_{jm}(\delta_{il}\cdot\delta_{jm} - \delta_{im}\cdot\delta_{jl}) =$$

$$= a_i\,\delta_{jm}^2 - a_j\cdot\delta_{jm}\cdot\delta_{im} = a_i\cdot\delta_{jm}^2 - a_i = a_i(\underbrace{\delta_{jm}^2}_{3} - 1) = 2\vec a$$

$$\mathrm{div}(\vec a\times\vec r) = \delta_{ik}\cdot\frac{\partial}{\partial x_i}(\vec a\times\vec r)_k = \delta_{ik}\cdot\frac{\partial}{\partial x_i}\varepsilon_{klm}\,a_l\cdot x_m =$$

$$= \delta_{ik}\cdot\varepsilon_{klm}\,a_l\cdot\delta_{im} = \underbrace{\varepsilon_{ili}\cdot a_l}_{0} = 0$$

$$\vec a\times(\vec b\times\vec c) = \vec b(\vec a\vec c) - \vec c(\vec a$$

$\varepsilon_{ijk}\cdot\varepsilon_{ijm} = \delta_{jj}\,\delta_{km} - \delta_{jm}\cdot\delta_{kj} = 3\,\delta_{km} - \delta_{km} = 2\,\delta_{km}$

$$\operatorname{div}\left(y(r)(\vec a\times\vec r)\right) = \delta_{ik}\cdot\frac{\partial}{\partial x_i}\left(y(r)(\vec a\times\vec r)_k\right) = \delta_{ik}\,\frac{\partial y(r)}{\partial x_i}\,(\vec a\times\vec r)_k +$$

$$+\ \delta_{ik}\cdot y(r)\cdot\frac{\partial}{\partial x_i}(\vec a\times\vec r)_k = \delta_{ik}\cdot\frac{\partial y}{\partial r}\cdot\frac{\partial r}{\partial x_i}\cdot\varepsilon_{klm}\cdot a_l\cdot x_m +$$

$$+\ \delta_{ik}\cdot y(r)\cdot\frac{\partial}{\partial x_i}\left(\varepsilon_{klm}\cdot a_l\cdot x_m\right) = \delta_{ik}\cdot y'(r)\cdot\frac{x_i}{r}\cdot\varepsilon_{klm}\cdot a_l\cdot x_m +$$

$$+\ y(r)\cdot\delta_{ik}\cdot\varepsilon_{klm}\cdot a_l\cdot\delta_{im} = y'(r)\cdot\frac{x_k}{r}\cdot\varepsilon_{klm}\,a_l\,x_m + y(r)\,\underset{0}{\underset{\parallel}{\varepsilon_{ili}}}\,a_l =$$

$$= y'(r)\,\frac{\vec r}{r}\,(\vec a\times\vec r) = \frac{y'(r)}{r}\cdot\vec a\,\underset{0}{\underset{\parallel}{(\vec r\times\vec r)}} = 0$$

$$\operatorname{rot}\left(y(r)(\vec a\times\vec r)\right)_i = \varepsilon_{ijk}\,\frac{\partial}{\partial x_j}\,y(r)\cdot(\vec a\times\vec r)_k =$$

$$= \varepsilon_{ijk}\cdot(\vec a\times\vec r)_k\cdot\frac{\partial y(r)}{\partial r}\cdot\underset{\frac{x_j}{r}}{\underset{\parallel}{\frac{\partial r}{\partial x_j}}} + \varepsilon_{ijk}\cdot y(r)\cdot\frac{\partial}{\partial x_j}\,\varepsilon_{klm}\cdot a_l\cdot x_m =$$

$$= \varepsilon_{ijk}\cdot\varepsilon_{klm}\cdot a_l\cdot x_m\cdot y'(r)\cdot\frac{x_j}{r} + \varepsilon_{ijk}\cdot\varepsilon_{klm}\cdot y(r)\,a_l\cdot\delta_{jm} =$$

$$= (\delta_{il}\,\delta_{jm} - \delta_{im}\cdot\delta_{jl})\,a_l\cdot x_m\cdot\frac{y'(r)}{r}\,x_j + \varepsilon_{ijk}\cdot\varepsilon_{ljk}\,y(r)\cdot a_l =$$

$$= \frac{y'(r)}{r}\,a_i\,x_m\,x_m - \frac{y'(r)}{r}\,x_i\,x_l\cdot a_l + 2\,\delta_{il}\,y(r)\,a_l =$$

$$= \frac{y'(r)}{r}\left(\vec a(\vec r\cdot\vec r) - \vec r(\vec a\cdot\vec r)\right) + 2y(r)\cdot\vec a = \vec a\left(2y(r) + y'(r)\cdot\vec r\right) -$$

$$- \frac{\vec r(\vec a\cdot\vec r)}{r}\cdot y'(r)$$

$$\operatorname{div}(\bar{r}\times(\bar{a}\times\bar{r}))_i = \delta_{ij}\frac{\partial}{\partial x_i}\cdot\varepsilon_{jkl}x_k\cdot\varepsilon_{lmn}a_m x_n =$$

$$= \delta_{ij}\varepsilon_{jkl}\varepsilon_{lmn}a_m\frac{\partial}{\partial x_i}x_k x_n = \delta_{ij}\varepsilon_{jkl}\varepsilon_{lmn}a_m\cdot$$

$$\cdot(\delta_{ik}x_n + \delta_{in}x_k) = \varepsilon_{ikl}\varepsilon_{lmn}a_m\delta_{ik}x_n +$$

$$+ \varepsilon_{ikl}\varepsilon_{lmn}a_m\delta_{in}x_k = \underbrace{\delta_{ik}\varepsilon_{ikl}}_{\varepsilon_{kkl}}\varepsilon_{lmn}\overset{0}{\cancel{a_m}}x_n +$$

$$+ \varepsilon_{nkl}\varepsilon_{lmn}a_m x_k = \underset{\overset{\parallel}{0}}{\varepsilon_{kkl}}\varepsilon_{lmn}a_m x_n + \varepsilon_{nkl}\varepsilon_{lmn}a_m x_k =$$

$$= \varepsilon_{nkl}\varepsilon_{nlm}a_m x_k = -\varepsilon_{nkl}\varepsilon_{nml}a_m x_k = -2\delta_{km}a_m x_k =$$

$$= -2(\bar{a}\cdot\bar{r})$$

$$\operatorname{rot}(\bar{r}\times(\bar{a}\times\bar{r}))_i = \varepsilon_{ijk}\frac{\partial}{\partial x_j}\cdot\varepsilon_{klm}x_l\cdot\varepsilon_{mnp}a_n x_p =$$

$$= \varepsilon_{ijk}\varepsilon_{klm}\varepsilon_{mnp}a_n(x_l\delta_{jp} + x_p\delta_{jl}) = \varepsilon_{ipk}\varepsilon_{klm}\varepsilon_{mnp}a_n x_l +$$

$$+ \varepsilon_{ilk}\varepsilon_{klm}\varepsilon_{mnp}a_n x_p = (\delta_{il}\delta_{pm} - \delta_{im}\delta_{pl})\varepsilon_{mnp}a_n x_l +$$

$$+ (-\delta_{im})\varepsilon_{mnp}a_n x_p = \varepsilon_{npm}\delta_{pm}a_n x_i - \varepsilon_{inp}a_n x_p -$$

$$-2\delta_{im}\varepsilon_{mnp}a_n x_p = \underset{\overset{\parallel}{0}}{\varepsilon_{nmm}}a_n x_i - \varepsilon_{inp}a_n x_p - 2\varepsilon_{inp}a_n x_p =$$

$$= -3(\bar{a}\times\bar{r}) = 3(\bar{r}\times\bar{a})$$

$$\operatorname{div}(\vec{a}\cdot\vec{r})\vec{r} = (\nabla\cdot(\vec{a}\cdot\vec{r})\cdot\vec{r})_i = \delta_{ik}\cdot\frac{\partial}{\partial x_i}((\vec{a}\cdot\vec{r})\cdot\vec{r})_k =$$

$$= \delta_{ik}\cdot\frac{\partial}{\partial x_i}\, a_k x_k x_k = \frac{\partial}{\partial x_i}\, a_k x_i x_k = a_k x_k \overbrace{\delta_{ii}}^{3} + a_k x_i \delta_{ik} =$$

$$= 3\vec{a}\cdot\vec{r} + \vec{a}\cdot\vec{r} = 4\,\vec{a}\cdot\vec{r}$$

44.

$$\operatorname{grad}(\vec{A}(r)\cdot\vec{r}) = (\nabla\vec{A}(r)\cdot\vec{r})_i = \frac{\partial}{\partial x_i}\delta_{ik} A(r)_k x_k =$$

$$= \frac{\partial}{\partial x_i} A(r)_i x_k = x_k\cdot\frac{\partial A(r)_i}{\partial r}\cdot\overbrace{\frac{\partial r}{\partial x_i}}^{\frac{x_i}{r}} + A(r)_i\cdot\delta_{ik} =$$

$$= x_k\cdot\frac{x_i}{r} A'(r)_i + A'(r)_k = \frac{\vec{r}(\vec{r}\,\vec{A}'(r))}{r} + \vec{A}(r)$$

$$(\operatorname{grad}(\vec{A}(r)\cdot\vec{B}(r)))_i = \frac{\partial}{\partial x_i}\delta_{ik} A(r)_k B(r)_k = \frac{\partial}{\partial x_i} A(r)_i\cdot B(r)_k =$$

$$= B(r)_k\cdot\frac{\partial A(r)_i}{\partial r}\cdot\overbrace{\frac{\partial r}{\partial x_i}}^{\frac{x_i}{r}} + A(r)_i\cdot\frac{\partial B(r)_k}{\partial r}\cdot\overbrace{\frac{\partial r}{\partial x_i}}^{\frac{x_i}{r}} =$$

$$= B(r)_k\cdot A'(r)_i\,\frac{x_i}{r} + A(r)_i\cdot B'(r)_k\,\frac{x_i}{r} = \vec{B}(r)\left(\frac{\vec{r}\cdot\vec{A}'(r)}{r}\right) +$$

$$+ \vec{B}'(r)\cdot\left(\frac{\vec{r}\cdot\vec{A}(r)}{r}\right) = \frac{\vec{r}}{r}\left(\vec{B}(r)\cdot\vec{A}'(r) + A(r)\cdot\vec{B}'(r)\right)$$

$x_i \delta_{ik} =$

$$\operatorname{div} y(r)\cdot \vec{A}(r) = \nabla (y(r)\cdot \vec{A}(r))_i =$$

$$= \delta_{ik} \frac{\partial}{\partial x_i}(y(r)\cdot A(r)_k) = \delta_{ik}\cdot A(r)_k \cdot \frac{\partial y(r)}{\partial r}\cdot \underbrace{\frac{\partial r}{\partial x_i}}_{=\frac{x_i}{r}} +$$

$$+ \delta_{ik}\cdot y(r)\cdot \frac{\partial A(r)_k}{\partial r}\cdot \underbrace{\frac{\partial r}{\partial x_i}}_{=\frac{x_i}{r}} = A(r)_i \cdot y'(r)\cdot \frac{x_i}{r} + y(r)\cdot A'(r)_k \cdot \frac{x_k}{r} =$$

$$= \frac{y'(r)}{r}(\vec{A}(r)\cdot \vec{r}) + \frac{y(r)}{r}\cdot(\vec{A}'(r)\cdot \vec{r})$$

$$\operatorname{rot} y(r)\cdot \vec{A}(r) = \varepsilon_{ijk}\frac{\partial}{\partial x_j}(y(r)\cdot A(r)_k) =$$

$$= y(r)\cdot \varepsilon_{ijk}\frac{\partial A(r)_k}{\partial r}\cdot\frac{\partial r}{\partial x_j} + A(r)_k\cdot \varepsilon_{ijk}\cdot\frac{\partial}{\partial x_j} y(r) =$$

$$= y(r)\cdot \varepsilon_{ijk}\cdot A'(r)_k \cdot \frac{x_j}{r} + A(r)_k\cdot \varepsilon_{ijk}\cdot y'(r)\cdot\frac{x_j}{r} =$$

$$= \frac{y(r)}{r}(\vec{r}\times \vec{A}'(r)) + \frac{y'(r)}{r}(\vec{r}\times\vec{A}(r))$$

$B(r)_k =$

$$(\vec{l}\cdot\nabla)\, y(r)\cdot \vec{A}(r) =$$

$$= (\vec{l}\cdot\nabla)\overset{\downarrow}{y}(r)\,\vec{A}(r) + (\vec{l}\cdot\nabla)\, y(r)\cdot \overset{\downarrow}{\vec{A}}(r) = (\vec{l}\cdot\nabla y(r))\cdot \vec{A}(r) +$$

$) +$

$$+ y(r)(\vec{l}\cdot\nabla)\vec{A}(r) = \vec{A}(r)\cdot(\vec{l}\, y'(r)\cdot\frac{\vec{r}}{r}) + y(r)\,\frac{\vec{A}'(r)}{r}\cdot(\vec{l}\cdot\vec{r}) =$$

$$= \frac{y'(r)}{r}(\vec{l}\cdot\vec{r})\cdot\vec{A}(r) + y(r)\,\frac{\vec{A}(r)}{r}\,(\vec{l}\cdot\vec{r})$$

Доза №2

№ 31

$$\oint_S \vec{n}\cdot\psi\cdot dS = \oint_S \psi\cdot d\vec{S} = \int_V \nabla\psi\, dV = \int_V \mathrm{grad}\,\psi\cdot dV$$

$$\oint_S (\vec{n}\times\vec{a})\,dS = \oint_S (d\vec{S}\times\vec{a}) = \int_V dV(\nabla\times\vec{a}) = \int_V \mathrm{rot}\,\vec{a}\cdot dV$$

$$\oint_S (\vec{n}\cdot\vec{b})\,\vec{a}\cdot dS = \oint_S (d\vec{S}\cdot\vec{b})\cdot\vec{a} = \int_V dV(\nabla\cdot\vec{b})\,\vec{a} =$$

$$\left\{(\nabla\cdot\vec{b})\,\vec{a} = (\nabla\cdot\overset{\downarrow}{\vec{b}})\,\vec{a} + (\nabla\vec{b})\,\overset{\downarrow}{\vec{a}} = (\vec{b}\,\nabla)\,\vec{a}\right\} = \int_V (\vec{b}\,\nabla)\,\vec{a}\,dV$$

№ 37.

$$\oint_S \vec{A}\cdot d\vec{S} = \int_V \mathrm{div}\,\vec{A}\cdot dV$$

$$\int_V (\vec{A}\cdot \mathrm{rot}\,\mathrm{rot}\,\vec{B} - \vec{B}\cdot\mathrm{rot}\,\mathrm{rot}\,\vec{A})\,dV = \oint_S [(\vec{B}\times\mathrm{rot}\,\vec{A}) - (\vec{A}\times\mathrm{rot}\,\vec{B})]\cdot d\vec{S}$$

$$\oint_S [(\vec{B}\times\mathrm{rot}\,\vec{A}) - (\vec{A}\times\mathrm{rot}\,\vec{B})]\cdot d\vec{S} = \int_V \mathrm{div}[(\vec{B}\times\mathrm{rot}\,\vec{A}) -$$

$$- (\vec{A}\times\mathrm{rot}\,\vec{B})]\,dV$$

$$\nabla(\vec{B}\times \mathrm{rot}\,\vec{A}) = \nabla(\overset{\downarrow}{\vec{B}}\times \mathrm{rot}\,\vec{A}) + \nabla(\vec{B}\times \mathrm{rot}\,\overset{\downarrow}{\vec{A}}) =$$

$$= \mathrm{rot}\,\vec{A}\cdot \mathrm{rot}\,\vec{B} - \vec{B}(\nabla\times(\nabla\times\vec{A})) = \mathrm{rot}\,\vec{A}\cdot \mathrm{rot}\,\vec{B} - \vec{B}\cdot \mathrm{rot}\,\mathrm{rot}\,\vec{A}$$

$$\nabla(\vec{A}\times \mathrm{rot}\,\vec{B}) = \mathrm{rot}\,\vec{A}\cdot \mathrm{rot}\,\vec{B} - \vec{A}\cdot \mathrm{rot}\,\mathrm{rot}\,\vec{B}$$

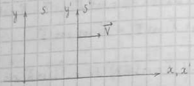
$$\int_V \mathrm{div}\left[(\vec{B}\times \mathrm{rot}\,\vec{A}) - (\vec{A}\times \mathrm{rot}\,\vec{B})\right] dV = \int_V (\cancel{\mathrm{rot}\,\vec{A}\cdot \mathrm{rot}\,\vec{B}} -$$

$$- \vec{B}\cdot \mathrm{rot}\,\mathrm{rot}\,\vec{A} - \cancel{\mathrm{rot}\,\vec{B}\cdot \mathrm{rot}\,\vec{A}} + \vec{A}\cdot \mathrm{rot}\,\mathrm{rot}\,\vec{B})\, dV =$$

$$= \int_V (-\vec{B}\cdot \mathrm{rot}\,\mathrm{rot}\,\vec{A} + \vec{A}\cdot \mathrm{rot}\,\mathrm{rot}\,\vec{B})\, dV$$

Доза №3

№ 548



условие: $x = x' = 0$, $t = t' = 0$

S: $x = x(t)$ - ?

S': $x' = x'(t)$ - ?

$$t' = \frac{t - \frac{V}{c^2}x}{\sqrt{1 - \frac{V^2}{c^2}}} \quad \text{т.к. } t = t' \quad t = \frac{t - \frac{V}{c^2}x}{\sqrt{1 - \frac{V^2}{c^2}}}$$

1) S: $t - \frac{V}{c^2}x = t\sqrt{1 - \frac{V^2}{c^2}}$

$$\frac{V}{c^2}x = t\left(1 - \sqrt{1 - \frac{V^2}{c^2}}\right)$$

$$x = \frac{c^2 t}{V}\left(1 - \sqrt{1 - \frac{V^2}{c^2}}\right)$$ - для S $x = x(t)$

2) S': $t = \frac{t' + \frac{V}{c^2}x'}{\sqrt{1 - \frac{V^2}{c^2}}}$ т.к. $t = t'$ $t' = \frac{t' + \frac{V}{c^2}x'}{\sqrt{1 - \frac{V^2}{c^2}}}$

$$t' + \frac{V}{c^2}x' = t'\sqrt{1 - \frac{V^2}{c^2}} \quad \Big| : t'$$

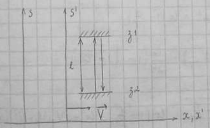
$$1 + \frac{V}{t' c^2} x' = \sqrt{1 - \frac{V^2}{c^2}}$$

$$\frac{V}{t' c^2} x' = \sqrt{1 - \frac{V^2}{c^2}} - 1 \qquad x' = \left(\sqrt{1 - \frac{V^2}{c^2}} - 1\right) \cdot \frac{c^2}{V} t' \qquad x' = x'(t)$$

мир. т. движется в S - вдоль x

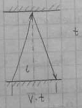
в S' - против x'

№ 578



в S': $c\tau = 2l$ $\tau = \frac{2l}{c}$

S:



$$ct = 2\sqrt{l^2 + \left(\frac{Vt}{2}\right)^2}$$

$$ct = \sqrt{4l^2 + V^2 t^2}$$

$$c^2 t^2 = 4l^2 + V^2 t^2$$

$$t^2 (c^2 - V^2) = 4l^2$$

$$t^2 = \frac{4l^2}{c^2\left(1-\frac{v^2}{c^2}\right)} \qquad t = \frac{\overset{=\tau}{\frac{2l}{c}}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\tau = t\cdot\sqrt{1-\frac{v^2}{c^2}}$$

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$$l_0 \longrightarrow \longleftarrow l_0$$

$$l_2' = l_0\cdot\frac{1}{\gamma} \qquad \Delta l = l_0\left(1-\frac{1}{\gamma}\right)$$

$$V = \frac{\Delta l}{\Delta t} = \frac{l_0}{\Delta t}\left(1-\frac{1}{\gamma}\right) = \frac{l_0}{\Delta t}\left(1-\sqrt{1-\frac{V^2}{c^2}}\right)$$

$$V\Delta t = l_0 - l_0\sqrt{1-\frac{V^2}{c^2}} \qquad l_0^2\left(1-\frac{V^2}{c^2}\right) = l_0^2 - 2V\Delta t l_0 + V^2(\Delta t)^2$$

$$V^2\left(\Delta t^2 + \frac{l_0^2}{c^2}\right) = \cancel{l_0^2} - \cancel{l_0^2} + 2\cancel{V}\cdot\Delta t l_0 \qquad V = \frac{2\Delta t l_0}{\Delta t^2 + \frac{l_0^2}{c^2}}$$

$$V = \frac{2l_0\Delta t}{(\Delta t)^2 + \frac{l_0^2}{c^2}} \qquad V(\Delta t)^2 + V\frac{l_0^2}{c^2} - 2l_0\cdot\Delta t = 0$$

$$D = 4l_0^2 - 4V^2\frac{l_0^2}{c^2} = 4l_0^2\left(1-\frac{V^2}{c^2}\right) > 0 \qquad \frac{V^2}{c^2} < 1 \qquad \left|\frac{V}{c}\right| < 1$$

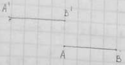
а) первый неподвижен - сначала лев. конец, потом правый

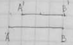
б) из симметрии для второго одновр-но

в) для лаб. сист. - одновременно

$$\frac{2ab}{a^2+b^2} = \frac{2ab}{(a-b)^2+2ab} \leq 1$$

549.





с т. зр. наблюдателя:

из т. B на B': она пройдет l_0 со ск. v_n

$$v_B = \frac{v_{B'} + v_n}{1 + \frac{v_{B'} v_n}{c^2}}$$

$v_n' = 0$ B' покоится в собств. с.о.

$t_{B0} = 12ч\,00$ часы A и B синхрониз.

$$t_A = t_B = 12ч\,00 + \frac{l_0}{v_n} = 12ч\,00 + \frac{8{,}64\cdot10^8}{24\cdot10^4} = 13ч\,00$$

$$(t_A - t_{A0})V = l_0$$

первонач. часы A и B' были синхр.

с учетом Лоренц. сокр:

$$\Delta t_{B'} = \Delta t_A \sqrt{1 - \frac{v_n^2}{c^2}} = \Delta t_A \cdot 0{,}6 = 36 \text{ мин}$$

$t_{B'} = 12ч\ 36$ мин

$$t_A = \frac{t_A' + \frac{v_n}{c^2}\cdot x_A'}{\sqrt{}} \qquad t_B = \frac{t_B' + \frac{v_n}{c^2}\cdot x_B'}{\sqrt{}}$$

$$t_{A'} - t_{B'} = \frac{v_n}{c^2}\left(x_{B'} - x_{A'}\right) = \frac{v_n}{c^2} \cdot l_0 = 38,4 \text{ мин}$$

$$t_{A'} = 38,4 \text{ мин} + 12,36 = 13ч\ 14,4 \text{ мин}$$

$$t_A = t_B = 13ч\ 00 \qquad t_{A'} = 13ч\ 14,4 \text{ мин} \qquad t_{B'} = 12ч\ 36 \text{ мин}$$

Doza № 4.

№ 554

$$\vec{r} = \frac{\vec{r}\,' + \vec{V}\cdot t' + \frac{1}{V^2}\cdot\vec{V}\times(\vec{V}\times\vec{r}\,')\left(1-\sqrt{1-\frac{V^2}{c^2}}\right)}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$t = \frac{t' + \frac{\vec{V}\cdot\vec{r}\,'}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}\,' + \vec{V}\cdot t' + \frac{1}{V^2}\cdot\vec{V}\times(\vec{V}\times d\vec{r}\,')\left(1-\sqrt{1-\frac{V^2}{c^2}}\right)}{dt' + \frac{\vec{V}\cdot d\vec{r}\,'}{c^2}} =$$

$$= \frac{\vec{v}\,' + \vec{V} + \frac{1}{V^2}\cdot\vec{V}\times(\vec{V}\times\vec{v}\,')\left(1-\sqrt{1-\frac{V^2}{c^2}}\right)}{1+\frac{\vec{V}\cdot\vec{v}\,'}{c^2}}$$

$\sqrt{}$ 560

Отн. скор-ть в лаб. сист

$V_2 = 2V = 1,8c$

S: $-V_S = \frac{-V-V}{1+\frac{V^2}{c^2}} = \frac{-2V}{1+\frac{V^2}{c^2}} = V\frac{(-2)}{1+0,81} =$

$= V\frac{-2}{1,81}$

$\frac{V_S}{c} = 0,9 \cdot \frac{-2}{1,81} = -\frac{1,8}{1,81} \approx -0,994$

$\frac{V_{S'}}{c} = 0,994$

1

$\vec{v}_1 = \vec{e}_x \cdot v$ $\quad \vec{v}_2 = -\vec{e}_x \cdot v$

$\vec{V} = \vec{v}, \quad V = v \qquad v'_{2x} = \frac{v_{2x} - V}{1 - \frac{V \cdot v_{2x}}{c^2}} = \frac{-v-v}{1+\frac{v^2}{c^2}}$

относительная скор. - это скорость движения частицы в той системе отсчета, в кот. др. покоится

$$V_x = \frac{V_x' + V}{1 + \frac{V_x' \cdot V}{c^2}}$$

$V_y = V_z = 0 \quad V_x = V, \quad V_x' = V'$

Доза №5.

№ 550

$l = 4$ года $\cdot c$

$v = \sqrt{0{,}9999}\, c$

$m = 10^4$ кг

$t = \frac{2 \cdot l}{v} = \frac{2 \cdot 4 \text{ года} \cdot \cancel{c}}{\underbrace{\sqrt{0{,}9999}}_{\approx 1} \cancel{c}} \approx 8$ лет

$T = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 = mc^2(\gamma - 1)$

$dt = \frac{dt'}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\quad dt' = dt \cdot \sqrt{1 - \frac{v^2}{c^2}} = 8$ лет $\cdot 364 \cdot 10^{-2} =$

≈ 29 дней ≈ 1 месяц

$T = 10^4 \cdot 9 \cdot 10^{16} \cdot \left(\frac{1}{\sqrt{1 - 0{,}9999}} - 1 \right) = 9 \cdot 10^{20} (10^2 - 1) =$

$= 891 \cdot 10^{20}$ Дж $= \frac{891 \cdot 10^{20}}{0{,}36 \cdot 10^4}$ Вт ч $= 2{,}475 \cdot 10^6$ кВт $\cdot$ ч

№ 556

док-ть ф-лу:

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{\sqrt{1-\frac{v'^2}{c^2}}\cdot\sqrt{1-\frac{V^2}{c^2}}}{1+\frac{\vec{v}'\cdot\vec{V}}{c^2}}$$

№ 557

где $\vec{v}$ и $\vec{v}'$ - скорости частицы в сист. S и S', $\vec{V}$ - скорость S' относ. S

$$v^2 = \frac{(\vec{v}'+\vec{V})^2 - (\vec{v}'\times\vec{V})^2\frac{1}{c^2}}{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2}$$

$$1-v^2 = 1-\frac{(\vec{v}'+\vec{V}) - (\vec{v}'\times\vec{V})^2\frac{1}{c^2}}{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2}$$

$$1-v^2 = \frac{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2 - (\vec{v}'+\vec{V}) + (\vec{v}'\times\vec{V})^2\frac{1}{c^2}}{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2}$$

$$\frac{v^2}{c^2} = \frac{\frac{(\vec{v}'+\vec{V})^2}{c^2} - \frac{(\vec{v}'\times\vec{V})^2}{c^4}}{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2}$$

$$(\vec{v}'\times\vec{V})^2 = v'^2V^2 - (\vec{v}'\cdot\vec{V})^2$$

$$1-\frac{v^2}{c^2} = \frac{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2 - \frac{(\vec{v}'+\vec{V})^2}{c^2} + \frac{(\vec{v}'\times\vec{V})^2}{c^4}}{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2} =$$

$$= \frac{1+\cancel{\frac{2\vec{v}'\cdot\vec{V}}{c^2}}+\cancel{\frac{(\vec{v}'\cdot\vec{V})^2}{c^4}} - \frac{v'^2}{c^2} - \frac{V^2}{c^2} - \cancel{\frac{2\vec{v}'\cdot\vec{V}}{c^2}} + \frac{v'^2\cdot V^2}{c^4} - \cancel{\frac{(\vec{v}'\cdot\vec{V})^2}{c^4}}}{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2} =$$

$$= \frac{1-\frac{v'^2}{c^2}-\frac{V^2}{c^2}+\frac{v'^2V^2}{c^4}}{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2} = \frac{\left(1-\frac{v'^2}{c^2}\right)\left(1-\left(\frac{V}{c}\right)^2\right)}{\left(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\right)^2}$$

← 362

$$\vec{v} = \frac{\vec{v'} + \vec{V} + (\gamma-1)\frac{\vec{V}}{V^2}\left[(\vec{v'}\cdot\vec{V}) + V^2\right]}{\gamma\left(1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}\right)} \qquad \left(1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}\right)$$

$$\frac{d\vec{v}}{dt'} = \frac{\vec{a'} + (\gamma-1)\frac{\vec{V}}{V^2}(\vec{a'}\cdot\vec{V})}{\gamma\left(1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}\right)} - \frac{\left(\vec{v'} + \vec{V} + (\gamma-1)\frac{\vec{V}}{V^2}\left[(\vec{v'}\cdot\vec{V}) + V^2\right]\right)\frac{\vec{a'}\cdot\vec{V}}{c^2}}{\gamma\left(1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}\right)^2}$$

$$\frac{d\vec{v}}{dt'} = \frac{1}{\gamma c^2\left(1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}\right)^2}\left[\vec{a'}\left(c^2 + (\vec{v'}\cdot\vec{V})\right) + \left((\gamma-1)\frac{c^2}{V^2}(\vec{a'}\cdot\vec{V})\left(1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}\right)\right.\right.$$

$$\left.\left. - (\gamma-1)\frac{1}{V^2}\left[\vec{v'}\cdot\vec{V} + V^2\right](\vec{a'}\cdot\vec{V})\right)\vec{V} + \vec{v'}\cdot(-\vec{a'}\cdot\vec{V}) - \vec{V}(\vec{a'}\vec{V})\right]$$

$$= \frac{1}{\gamma c^2\left(1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}\right)^2}\left[\vec{a'}\left(c^2 + (\vec{v'}\cdot\vec{V})\right) + \vec{V}(\gamma-1)(\vec{a'}\cdot\vec{V})\left(\frac{c^2}{V^2} + \frac{\vec{v'}\cdot\vec{V}}{V^2} - \frac{\vec{v'}\vec{V}}{V^2}\right.\right.$$

$$\left. - \vec{v'}(\vec{a'}\vec{V}) - \vec{V}(\vec{a'}\vec{V})\right]$$

$$= \frac{1}{\gamma c^2\left(1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}\right)^2}\left[\vec{a'}\left(c^2 + \vec{v'}\vec{V}\right) + \vec{V}(\vec{a'}\vec{V})(\gamma-1)\left(\frac{c^2}{V^2} - 1\right) - \right.$$

$$\left. - \vec{v'}(\vec{a'}\vec{V}) - \vec{V}(\vec{a'}\vec{V})\right] =$$

$$S = 1 + \frac{\vec{v'}\cdot\vec{V}}{c^2}$$

$$\frac{d\vec{v}}{dt'} = \frac{1}{\gamma S^2}\cdot\frac{1}{c^2}\Big[\vec{a}'(c^2+\vec{v}'\cdot\vec{V}) + \vec{V}(\vec{a}'\cdot\vec{V})(\gamma-1)\frac{c^2-V^2}{V^2} -$$

$$-\vec{v}'(\vec{a}'\vec{V}) - \vec{V}(\vec{a}'\vec{V})\Big]$$

$$\frac{dt'}{dt} = \frac{1}{\frac{dt}{dt'}} = \frac{\sqrt{1-\frac{V^2}{c^2}}}{1+\frac{\vec{V}\,\vec{v}'}{c^2}} = \frac{1}{\gamma S}$$

$$\frac{d\vec{v}}{dt} = \frac{1}{\gamma^2 S^3}\Big[\vec{a}'\underbrace{\Big(1+\frac{\vec{v}'\cdot\vec{V}}{c^2}\Big)}_{S} + \vec{V}(\vec{a}'\vec{V})\Big((\gamma-1)\cdot\frac{1}{V^2}\Big(1-\frac{V^2}{c^2}\Big)-\frac{1}{c^2}\Big) -$$

$$-\vec{v}'\,\frac{\vec{a}'\cdot\vec{V}}{c^2}\Big]$$

$$\frac{d\vec{v}}{dt} = \frac{\vec{a}'}{\gamma^2 S^2} - \frac{\vec{v}'(\vec{a}'\vec{V})}{\gamma^2 S^3 c^2} + \vec{V}(\vec{a}'\vec{V})\frac{1}{\gamma^2 S^3}\left(\left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}-1\right)\left(1-\frac{V^2}{c^2}\right)-\frac{V^2}{c^2}\right)\frac{1}{V^2}\Big) =$$

$$\Rightarrow \frac{d\vec{v}}{dt} = \frac{\vec{a}'}{\gamma^2 S^2} - \frac{\vec{v}'(\vec{a}'\vec{V})}{\gamma^2 S^3 c^2} + \vec{V}(\vec{a}'\vec{V})\frac{1}{\gamma^2 S^3}\left[\left(\left(\frac{1}{\sqrt{1-\frac{V^2}{c^2}}}-1\right)\left(1-\frac{V^2}{c^2}\right)-\frac{V^2}{c^2}\right)\frac{1}{V^2}\right]$$

$$\frac{1}{V^2}\frac{\left(1-\sqrt{1-\frac{V^2}{c^2}}\right)\left(1-\frac{V^2}{c^2}\right)-\frac{V^2}{c^2}\sqrt{1-\frac{V^2}{c^2}}}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{1}{V^2}\gamma\cdot\left(1-\frac{V^2}{c^2}-\sqrt{1-\frac{V^2}{c^2}}+\cancel{\sqrt{1-\frac{V^2}{c^2}}\frac{V^2}{c^2}}-\cancel{\frac{V^2}{c^2}\cdot\sqrt{1-\frac{V^2}{c^2}}}\right)$$

$$= \left\{\begin{aligned} &\sqrt{1-\frac{V^2}{c^2}}-1 = \frac{1}{\gamma}-1 = \frac{1-\gamma}{\gamma} \qquad \gamma = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} \\ &1-\frac{V^2}{c^2} = \frac{1}{\gamma^2} \Rightarrow \frac{V^2}{c^2} = 1-\frac{1}{\gamma^2} \end{aligned}\right\} = \frac{\gamma}{V^2}\left(1-\frac{V^2}{c^2}-\sqrt{1-\frac{V^2}{c^2}}\right) =$$

$$= \frac{\gamma}{V^2}\left(\frac{1}{\gamma^2}-\frac{1}{\gamma}\right) = \frac{(1-\gamma)}{V^2\gamma}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$s = 1 + \frac{\vec{v}' \cdot V}{c^2}$$

$$\frac{d\vec{v}}{dt} = \frac{\vec{a}'}{\gamma^2 s^2} - \frac{\vec{v}'(\vec{a}' \cdot \vec{V})}{\gamma^2 s^3 c^2} - \frac{\vec{V}(\vec{a}' \cdot \vec{V})(\gamma - 1)}{V^2 \gamma^3 s^3}$$

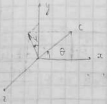
$$\frac{d\vec{v}}{dt} = \frac{1 - \frac{V^2}{c^2}}{\left(1 + \frac{\vec{v}' \cdot \vec{V}}{c^2}\right)^3} \left(\dot{\vec{v}}'\left(1 + \frac{\vec{v}' \cdot \vec{V}}{c^2}\right) - \frac{(\dot{\vec{v}}' \cdot \vec{V})\vec{v}'}{c^2} - \frac{\vec{V}(\dot{\vec{v}}' \cdot \vec{V})}{V^2}\left(1 - \sqrt{1 - \frac{V^2}{c^2}}\right.\right.$$

Doza №6

572

$d\Omega$

$d\Omega'$?



$$d\Omega = \sin\theta \, d\theta \, d\varphi$$

$$d\Omega' = \sin\theta' \, d\theta' \, d\varphi'$$

$$v_x = \frac{v_x' + V}{1 + \frac{V v_x'}{c^2}}$$

$$v_y = \frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V v_x'}{c^2}} v_y'$$

$$v_z = \frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V v_x'}{c^2}} v_z'$$

$v_x = c \cdot \cos\theta$ $\qquad$ $v_x' = c \cdot \cos\theta'$

$v_y = c \sin\theta \cdot \cos\varphi$ $\qquad$ $v_y' = c \cdot \sin\theta' \cdot \cos\varphi'$

$v_z = c \cdot \sin\theta \cdot \sin\varphi$ $\qquad$ $v_z' = c \cdot \sin\theta' \cdot \sin\varphi'$

① $$c \cdot \cos\theta = \frac{c \cdot \cos\theta' + V}{1 + \frac{V \cancel{c} \cos\theta'}{c^2}}$$

② $$\cancel{c} \sin\theta \cdot \cos\varphi = \frac{\sqrt{1 - \frac{V^2}{c^2}} \, \cancel{c} \sin\theta' \cdot \cos\varphi'}{1 + \frac{V \cdot \cancel{c} \cdot \cos\theta'}{c^2}}$$

③ $$\cancel{c} \cdot \sin\theta \cdot \sin\varphi = \frac{\sqrt{1 - \frac{V^2}{c^2}} \, \cancel{c} \cdot \sin\theta' \cdot \sin\varphi'}{1 + \frac{V \cdot \cos\theta'}{c}}$$

① $\cos\theta = \dfrac{\cos\theta' + \frac{V}{c}}{1 + \frac{V}{c}\cdot\cos\theta'}$

$$\cos\theta + \frac{V}{c}\cos\theta\cdot\cos\theta' = \cos\theta' + \frac{V}{c}$$

$$\cos\theta' = \frac{\cos\theta - \frac{V}{c}}{1 - \frac{V}{c}\cdot\cos\theta}$$

② $\dfrac{\cos\varphi}{\cos\varphi'} = \dfrac{\sqrt{1-\frac{V^2}{c^2}}\cdot\frac{\sin\theta'}{\sin\theta}}{1+\frac{V}{c}\cos\theta'} = \dfrac{\sqrt{1-\frac{V^2}{c^2}}\sqrt{1-\frac{\cos^2\theta - 2\frac{V}{c}\cos\theta + \ldots}{\left(1-\frac{V}{c}\cdot\cos\theta\right)^2}}}{\sin\theta\left(1+\frac{V}{c}\left(\frac{\cos\theta-\frac{V}{c}}{1-\frac{V}{c}\cos\theta}\right)\right)} =$

$$= \frac{\sqrt{1-\frac{V^2}{c^2}}\sqrt{1-\cancel{2\frac{V}{c}\cos\theta}+\frac{V^2}{c^2}\cdot\cos^2\theta-\cos^2\theta+\cancel{2\frac{V}{c}\cos\theta}-\frac{V^2}{c^2}}}{\sin\theta\cdot\left(1-\cancel{\frac{V}{c}\cos\theta}+\cancel{\frac{V}{c}\cdot\cos\theta}-\frac{V^2}{c^2}\right)} =$$

$$= \frac{\sqrt{1-\cos^2\theta+\left(-\frac{V^2}{c^2}\right)\left(1-\cos^2\theta\right)}}{\sin\theta\sqrt{1-\frac{V^2}{c^2}}} = 1$$

$\varphi = \varphi' \qquad d\varphi = d\varphi'$

$d\Omega' = \sin\theta'\, d\theta' \cdot d\varphi'$

продиф. слева по $\dfrac{d}{d\theta'}$, справа по $\dfrac{d}{d\theta}$

$$-\sin\theta'\, d\theta' = -\frac{\sin\theta\cdot d\theta}{1-\frac{V}{c}\cos\theta} - \frac{\left(\cos\theta-\frac{V}{c}\right)\frac{V}{c}\cdot\sin\theta\cdot d\theta}{\left(1-\frac{V}{c}\cos\theta\right)^2}$$

$\sin\theta'$

$= \dfrac{1-\ldots}{(1-\ldots}$

$d\Omega$

$d\Omega$

$2\frac{v}{c}\cos\theta + \frac{v^2}{c^2}$

$\cos\theta)^2$

$\theta - \frac{v}{c}$

$\frac{v}{c}\cos\theta$

$\frac{v^2}{c^2}$

$$\sin\theta' \, d\theta' = \frac{\sin\theta - \frac{v}{c}\cancel{\sin\theta\cos\theta} + \frac{v}{c}\cancel{\cos\theta\sin\theta} - \frac{v^2}{c^2}\sin\theta}{\left(1 - \frac{v}{c}\cos\theta\right)^2} \, d\theta =$$

$$= \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v}{c}\cos\theta\right)^2} \sin\theta \, d\theta$$

$$d\Omega' = \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v}{c}\cos\theta\right)^2} \cdot \sin\theta \, d\theta \cdot d\varphi \qquad d\Omega = \sin\theta \cdot d\theta \, d\varphi$$

$$d\Omega' = \frac{1 - \frac{v^2}{c^2}}{\left(1 - \frac{v}{c}\cos\theta\right)^2} \, d\Omega$$

№ 584

а) $\vec{\dot v}'(\vec{\dot v})$ - ?

$\vec v = \vec\tau(t)\cdot v_0$ по напр.

б) $\vec v = v(t)\cdot\vec\tau_0$

из зад № 583:

$$w_i^2 = -\frac{1}{c^4\left(1-\frac{v^2}{c^2}\right)^3}\cdot\left\{\dot{\vec v}^2 - \frac{(\vec v\times\dot{\vec v})^2}{c^2}\right\}$$

$\downarrow$

inv

$w_i^2 = w_i'^2$ $\quad \vec v' = 0$ (мгновенно сопутств. сист. отсч.)

$$-\frac{1}{c^4\left(1-\frac{v^2}{c^2}\right)^3}\left\{\dot{\vec v}^2 - \frac{(\vec v\times\dot{\vec v})^2}{c^2}\right\} = -\frac{\dot{\vec v}'^2}{c^4} \Rightarrow$$

$$\dot{\vec v}'^2 = \frac{1}{\left(1-\frac{v^2}{c^2}\right)^3}\left\{\dot{\vec v}^2 - \frac{(\dot{\vec v}\times\vec v)^2}{c^2}\right\}$$

а) $\vec v = \vec\tau(t)\cdot v_0$ $\quad \vec v \perp \dot{\vec v}$ $\Rightarrow$

$$\Rightarrow \dot{\vec v}'^2 = \gamma^6\left\{\dot{\vec v}^2 - \dot{\vec v}^2\cdot\frac{\vec v^2}{c^2} + \cancel{\left(\frac{\vec v\cdot\dot{\vec v}}{c}\right)^2}\right\} =$$

$$= \gamma^6\cdot\dot{\vec v}^2\underbrace{\left\{1-\frac{v^2}{c^2}\right\}}_{=\frac{1}{\gamma^2}} = \frac{1}{\left(1-\frac{v^2}{c^2}\right)^2}\dot{\vec v}^2$$

$$\dot{\vec v}' = \frac{\dot{\vec v}}{1-\frac{v^2}{c^2}}$$

8) $\dot{\vec{v}} = \vec{v_0} \cdot \delta(t) \qquad \vec{v} \parallel \dot{\vec{v}} \qquad \dot{\vec{v}}' = \dfrac{1}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \dot{\vec{v}}$

№ 569

$$\cos\alpha' = \frac{(\vec{v}_1 - \vec{V})(\vec{v}_2 - \vec{V}) - \frac{1}{c^2}(\vec{v}_1 \times \vec{V})(\vec{v}_2 \times \vec{V})}{\sqrt{(\vec{v}_1 - \vec{V})^2 - \frac{1}{c^2}(\vec{v}_1 \times \vec{V})^2} \sqrt{(\vec{v}_2 - \vec{V})^2 - \frac{1}{c^2}(\vec{v}_2 \times \vec{V})^2}}$$

wz zag. 562

$$((\vec{a} \times \vec{b})(\vec{c} \times \vec{b}))_i = \varepsilon_{ijk} a_j b_k \varepsilon_{ilm} c_l b_m =$$

$$= a_j c_l b_k b_m (\delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}) = a_l c_l b_m b_m - a_m b_m c_l b_l = (\vec{a}\,\vec{c})b^2 - (\vec{a}\cdot\vec{b})(\vec{c}\cdot\vec{b})$$

$V^2 \to c^2$

$$\cos\alpha' = \frac{(\vec{v}_1 - \vec{V})(\vec{v}_2 - \vec{V}) - \frac{1}{c^2}\left(V^2 \vec{v}_1 \cdot \vec{v}_2 - (\vec{v}_2 \cdot \vec{V})(\vec{v}_1 \cdot \vec{V})\right)}{\sqrt{(\vec{v}_1 - \vec{V})^2 - \frac{1}{c^2}(\vec{v}_1 \times \vec{V})^2} \cdot \sqrt{(\vec{v}_2 - \vec{V})^2 - \frac{1}{c^2}(\vec{v}_2 \times \vec{V})^2}} =$$

$$= \frac{\vec{v}_1 \cdot \vec{v}_2 - \vec{V}(\vec{v}_1 + \vec{v}_2) + V^2 - \frac{1}{c^2}\left(V^2 \vec{v}_1 \cdot \vec{v}_2 - (\vec{v}_1 \cdot \vec{V})(\vec{v}_2 \cdot \vec{V})\right)}{\sqrt{\left(c - \frac{\vec{v}_1 \cdot \vec{V}}{c}\right)^2} \cdot \sqrt{\left(c - \frac{\vec{v}_2 \cdot \vec{V}}{c}\right)^2}} =$$

$$(\vec{v}_1 \times \vec{V})^2 = v_1^2 V^2 - (\vec{v}_1 \cdot \vec{V})^2 \quad \Big| \; V \to c \quad V^2 = c^2$$

$$\cancel{v_1^2} - 2\vec{v}_1 \cdot \vec{V} + c^2 - \cancel{v_1^2} + \frac{(\vec{v}_1 \cdot \vec{V})^2}{c^2} = \left(c - \frac{\vec{v}_1 \cdot \vec{V}}{c}\right)^2$$

$$= \frac{c^2 - \vec{v}_1 \cdot \vec{V} - \vec{v}_2 \cdot \vec{V} + \frac{1}{c^2}(\vec{v}_1 \cdot \vec{V})(\vec{v}_2 \cdot \vec{V})}{\sqrt{\left(c - \frac{\vec{v}_1 \cdot \vec{V}}{c}\right)^2} \sqrt{\left(c - \frac{\vec{v}_2 \cdot \vec{V}}{c}\right)^2}}$$

cont.

$\Rightarrow$

$v^2 - \left(\dot{\vec{v}} \times \frac{\vec{v}}{c}\right)^2$

n 573

$$\frac{dN}{d\Omega'} \qquad V \to c$$

$$d\Omega = \frac{1 - \frac{v^2}{c^2}}{\left(1 + \frac{v}{c}\cos\theta'\right)^2} \, d\Omega'$$

$$\frac{dN}{d\Omega'} = \frac{dN}{d\Omega} \frac{d\Omega}{d\Omega'} = \frac{N_0}{4\pi} \cdot \frac{1 - \frac{v^2}{c^2}}{\left(1 + \frac{v}{c}\cos\theta'\right)^2} =$$

$$V \to c \qquad \frac{dN}{d\Omega'} \to 0 \quad \text{if} \quad \theta' \neq \pi$$

$$\frac{dN}{d\Omega'} \to \infty \quad \text{if} \quad \theta' = \pi$$

Doza №7

№ 622

$\vec{v}(\vec{p})$ - ?

$$\vec{v} = \frac{c^2\vec{p}}{\mathcal{E}_\kappa}$$

$$\vec{p} = \frac{m\vec{v}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \mathcal{E} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = c\sqrt{p^2+m^2c^2}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{mc}{\sqrt{p^2+m^2c^2}} \qquad \vec{p} = \frac{\cancel{m}\vec{v}\sqrt{p^2+m^2c^2}}{\cancel{m}c}$$

$$\vec{v} = \frac{c\cdot\vec{p}}{\sqrt{p^2+m^2c^2}}$$

№ 623

$m,\ \mathcal{E}$

v - ?

а) $\frac{v}{c} \ll 1$

б) $\frac{v}{c} \to 1$

$$\mathcal{E} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \frac{m^2c^4}{\mathcal{E}^2} \qquad \frac{v}{c} = \sqrt{1-\left(\frac{mc^2}{\mathcal{E}}\right)^2}$$

$$v = c\sqrt{1-\left(\frac{mc^2}{\mathcal{E}}\right)^2}$$

а) $v \ll c$

$$\mathcal{E} = mc^2\left(1+\frac{v^2}{2c^2}\right) \qquad \frac{v^2}{2c^2} = \frac{\mathcal{E}}{mc^2}-1 \qquad v = \sqrt{2}\,c\sqrt{\frac{\mathcal{E}}{mc^2}-1} =$$

$$= \sqrt{2}\cdot\sqrt{\frac{\mathcal{E}-mc^2}{m}}$$

б) $v \to c$

$\left(\frac{mc^2}{\varepsilon}\right)^2 \to 0 \qquad v = c\left(1 - \frac{1}{2}\left(\frac{mc^2}{\varepsilon}\right)^2\right)$

№ 636 $\qquad \beta = \frac{v}{c}$

$dw = \frac{\partial w}{\partial \Omega'} \cdot d\Omega'$

$\frac{dw}{d\Omega'} = \frac{1}{4\pi}$ — сферич. симметрия в сист. Π'

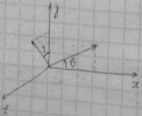
$1 = \frac{\partial w}{\partial \Omega'} \cdot 4\pi$

$$\frac{dw}{d\Omega} = \frac{1}{4\pi} \cdot \frac{1-\beta^2}{(1-\beta\cdot\cos\theta)^2}$$

Из 635: $\quad f = \frac{N_{пер}}{N_{зад}} \qquad \varepsilon(f) - ?$

$N_{пер}$ – число γ-квантов, испуск. в переднюю полусферу

$N_{зад}$ – в заднюю



$$N_{\substack{пер \\ (зад)}} = \int \frac{dN}{d\Omega} \cdot d\Omega = \int N_0 \cdot \frac{dw}{d\Omega} \cdot d\Omega$$

$$\Rightarrow N_{пр.} = \frac{N_0}{4\pi}(1-\beta^2)\int_0^{2\pi} d\varphi \cdot \int_0^{\pi/2} \frac{\sin\theta \cdot d\theta}{(1-\beta\cdot\cos\theta)^2} = \frac{N_0(1-\beta^2)}{2\beta}\cdot$$

$$\int_0^{\frac{\pi}{2}} \frac{d(1-\beta\cdot\cos\theta)}{(1-\beta\cdot\cos\theta)^2} = -\frac{N_0(1-\beta^2)}{2\beta}\left(1-\frac{1}{1-\beta}\right) = \frac{N_0(1-\beta)^2}{2(1-\beta)}$$

$$N_{зад} = -\frac{N_0}{4\pi}(1-\beta^2)\cdot\int_{\frac{\pi}{2}}^{\pi} \frac{\sin\theta \cdot d\theta}{(1-\beta\cdot\cos\theta)^2} =$$

$$= -\frac{N_0(1-\beta^2)}{2\beta}\cdot\left(\frac{1}{1+\beta}-1\right) = \frac{N_0(1-\beta^2)}{2(1+\beta)}$$

N_0 - общее число γ-квантов

$$f = \frac{N_{пр}}{N_{зад}} = \frac{1+\beta}{1-\beta} \Rightarrow f - f\beta = 1+\beta \quad \beta = \frac{f-1}{f+1}$$

$$\mathcal{E} = \frac{mc^2}{\sqrt{1-\beta^2}} = \frac{mc^2(f+1)}{\sqrt{1-\left(\frac{f-1}{f+1}\right)^2}} = \frac{mc^2(f+1)}{\sqrt{\cancel{f^2}+2f+\cancel{1}-\cancel{f^2}-\cancel{1}+2f}} =$$

$$= \frac{mc^2(f+1)}{2\sqrt{f}}$$

$$\mathcal{E} = \frac{mc^2(f+1)}{2\sqrt{f}} \quad f = \frac{1+\beta}{1-\beta}$$

626

Доза №8

№ 625

разн. пот. V

$v_0 = 0$

$v - ?$

$$\frac{dE}{dt} = 0$$

по з-ну сохр. эн: $\mathcal{E}_1 = mc^2 + e\varphi_1 = \mathcal{E}_2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + e\varphi_2$

$$e(\varphi_2 - \varphi_1) = mc^2\left(1 - \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2 + e(\varphi_1 - \varphi_2)$$

if $e>0$ то для ускор. необх. $\varphi_1 > \varphi_2$

if $e<0 \Rightarrow \varphi_1 < \varphi_2$

$$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2 + |eV| \qquad 1 - \frac{v^2}{c^2} = \left(\frac{mc^2}{mc^2+|eV|}\right)^2 \Rightarrow$$

$$v = c\cdot\sqrt{1-\left(\frac{mc^2}{mc^2+|eV|}\right)^2} = c\sqrt{\frac{\cancel{m^2c^4} + 2mc^2|eV| + (eV)^2 - \cancel{m^2c^4}}{(mc^2+|eV|^2)}} =$$

$$= \sqrt{\frac{2mc^2|eV| + |eV|^2}{m^2c^2\left(1+\frac{|eV|}{mc^2}\right)^2}} = \sqrt{\frac{2|eV|}{m}\cdot\frac{1+\frac{|eV|}{2mc^2}}{\left(1+\frac{|eV|}{mc^2}\right)^2}} = v$$

a) $v \ll c$ $$mc^2\left(1+\frac{1}{2}\frac{v^2}{c^2}\right) = mc^2 + |eV|$$

$$\cancel{1} + \frac{1}{2}\frac{v^2}{c^2} = \cancel{1} + \frac{|eV|}{mc^2} \qquad \frac{2|eV|}{m} = v^2 \Rightarrow v = \sqrt{\frac{2|eV|}{m}}$$

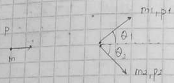
б) $v \to c$ $$\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2 + |eV|$$

$|eV| \to \infty$

$$v = c\sqrt{1-\left(\frac{mc^2}{mc^2}\frac{1}{\left(1+\frac{|eV|}{mc^2}\right)}\right)^2} \approx c\left(1-\frac{1}{2}\cdot\frac{1}{\left(1+\frac{|eV|}{mc^2}\right)^2}\right) =$$

$$= c\,\frac{\left(2-1+\frac{4|eV|}{mc^2}+2\frac{|eV|^2}{(mc^2)^2}\right)}{2\left(1+\frac{|eV|}{mc^2}\right)^2} \approx c\cdot\frac{(eV)^2}{(mc^2+|eV|)^2}$$

№ 641



$m_1 - ?$

з-н сохр. 4-черн. импульса

$p^i = p_1^i + p_2^i \qquad m^2c^2 = m_1^2c^2 + m_2^2c^2 + 2p_1^i p_{2i} =$

$= m_1^2c^2 + m_2^2c^2 + 2\dfrac{\mathcal{E}_1 \cdot \mathcal{E}_2}{c^2} - 2\vec{p_1}\cdot\vec{p_2}$

$\mathcal{E}_1 = c\sqrt{p_1^2 + m_1^2c^2}$

$\mathcal{E}_2 = c\cdot\sqrt{p_2^2 + m_2^2c^2}$

$p_1^i = p^i - p_2^i \qquad m_1^2c^2 = m^2c^2 + m_2^2\cdot c^2 - 2\cdot\dfrac{\mathcal{E}\cdot\mathcal{E}_2}{c^2} + 2\vec{p}\cdot\vec{p_2} =$

$= (m^2 + m_2^2)c^2 - 2\sqrt{(p_2^2 + m_2^2c^2)(p^2 + m^2c^2)} + 2pp_2\cdot\cos\theta_2$

№ 674

доказать невозможность:

$e^- \to e^- + \gamma$

$p_e^2 = \tilde{p}_e^2 + p_\gamma^2$

$m_e^2 c^2 = m_e^2 c^2 + 2\frac{\mathcal{E}_e \mathcal{E}_\gamma}{c^2} - 2\vec{p}_\gamma \vec{p}_e \Rightarrow \vec{p}_\gamma \cdot \vec{p}_e = \frac{\mathcal{E}_e \mathcal{E}_\gamma}{c^2}$

$p_e \cdot \cos\theta = \frac{\mathcal{E}_e}{c} = \sqrt{p_e^2 + m_e^2 c^2} > p_e \Rightarrow$

$p_e \cdot \cos\theta > p_e \Rightarrow \cos\theta > 1$ невозможно.

№ 534

$$\gamma + \left(\begin{array}{c}\text{ядро с} \\ \text{массой } m_1\end{array}\right) \longrightarrow e^+ + e^- + (\text{ядро})$$

доказать, что это возможно if $m_1 \neq 0$ T_0 - ?

Найти порог T_0 рождения пары (электрон-позитрон)

M - общ. масса

$p_\gamma^2 + p_1^2 = P^2$ $\quad m_1^2c^2 + 2\frac{\varepsilon m_1 c^2}{c^2} = M^2c^2$

$M = 2m_e + m_1$ $\quad v_e$ после вылета $= 0$

(порог реакции)

$\varepsilon = T_0 + \cancel{m_\gamma c^2} = T_0$ $\quad \varepsilon_\gamma = \cancel{mc^2} + T_0$

$$T_0 = \frac{c^2}{2m_1}(M^2 - m_1^2) = \frac{c^2}{2m_1}(4m_e^2 + 4m_e m_1) =$$

$= 2c^2 m_e\left(1 + \frac{m_e}{m_1}\right)$ при $m_1 \to 0$ $T_0 \to \infty$ (это невозможно)

$$T_0 = 2m_ec^2\left(1 + \frac{m_e}{m_1}\right)$$

↓ 534, 626 уст.

Доза № 9

№ 564

Используя формулу задачи № 562

$$\dot{\vec{v}} = \frac{1-\frac{V^2}{c^2}}{\left(1+\frac{\vec{V}\vec{v}'}{c^2}\right)^3}\left(\left(1+\frac{\vec{V}\vec{v}'}{c^2}\right)\dot{\vec{v}}' - \frac{\vec{V}\cdot\dot{\vec{v}}'}{c^2}\vec{v}' - \left(1-\sqrt{1-\frac{V^2}{c^2}}\right)\frac{\vec{V}(\dot{\vec{v}}'\vec{V})}{V^2}\right)$$

или

$$\dot{\vec{v}}' = \frac{1-\frac{V^2}{c^2}}{\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)^3}\cdot\left(\left(1-\frac{\vec{V}\cdot\vec{v}}{c^2}\right)\dot{\vec{v}} + \frac{\vec{V}\cdot\dot{\vec{v}}}{c^2}\cdot\vec{v} - \left(1-\sqrt{1-\frac{V^2}{c^2}}\right)\frac{\vec{V}\cdot\dot{\vec{v}}}{V^2}\cdot\vec{V}\right)$$

учитывая, что $\vec{V}=\vec{v}$, $\vec{v}'=0$

$$\dot{\vec{v}}' = \frac{1}{\left(1-\frac{v^2}{c^2}\right)^2}\left(\left(1-\frac{v^2}{c^2}\right)\dot{\vec{v}} + \frac{(\vec{v}\cdot\dot{\vec{v}})}{c^2}\vec{v} - \frac{(\vec{v}\cdot\dot{\vec{v}})}{v^2}\vec{v} + \sqrt{1-\frac{v^2}{c^2}}\cdot\frac{(\vec{v}\cdot\dot{\vec{v}})}{v^2}\vec{v}\right) =$$

$$= \frac{1}{\left(1-\frac{v^2}{c^2}\right)^2}\left(\left(1-\frac{v^2}{c^2}\right)\dot{\vec{v}} + \left(\frac{1}{c^2}-\frac{1}{v^2}+\frac{\sqrt{1-\frac{v^2}{c^2}}}{v^2}\right)(\vec{v}\cdot\dot{\vec{v}})\,\vec{v}\right)$$

$$\dot{\vec{v}}' = \frac{1}{\left(1-\frac{v^2}{c^2}\right)^2}\left(\left(1-\frac{v^2}{c^2}\right)\dot{\vec{v}} + \left(\sqrt{1-\frac{v^2}{c^2}}-1+\frac{v^2}{c^2}\right)\frac{(\vec{v}\cdot\dot{\vec{v}})}{v^2}\vec{v}\right) \qquad v\,\dot{v}$$

$$\frac{\dot{\vec{v}}}{1-\frac{v^2}{c^2}} + \sqrt{1-\frac{v^2}{c^2}}\,\dot{v} -$$

а) $\vec{v} = \tau(t)\cdot\vec{v}_0$ $\qquad \dot{\vec{v}}' = \dfrac{\dot{\vec{v}}}{1-\frac{v^2}{c^2}}$

$$\frac{(\vec{v}\cdot\dot{\vec{v}})}{v^2}\vec{v} =$$

б) $\vec{v} = v(t)\cdot\vec{\tau}_0$ $\qquad \dot{\vec{v}}' = \dfrac{1}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}\cdot\dot{\vec{v}}$

$$= \frac{\cancel{v}\,\dot{v}}{v^2}\cdot v\,\vec{\tau}_0 =$$

$\vec{v} \parallel \dot{\vec{v}}$

№ 626

а) $T = 300$ эВ, e

б) $T = 300$ МэВ, p, e

в) $T = 680$ МэВ, p

г) $T = 10$ ГэВ, p

v - ?

$$T = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{T}{mc^2} + 1 \qquad \sqrt{1-\frac{v^2}{c^2}} = \frac{mc^2}{T+mc^2}$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{mc^2}{T+mc^2}\right)^2 \qquad v = c\sqrt{1-\left(\frac{mc^2}{T+mc^2}\right)^2}$$

$m_e c^2 = 0{,}511$ МэВ

$m_p c^2 = 938{,}2796$ МэВ

а) $$v = 3\cdot 10^8 \cdot \sqrt{1-\left(\frac{0{,}51\cdot 10^6}{300+0{,}51\cdot 10^6}\right)^2} = c\sqrt{1-\left(\frac{1}{\frac{300}{0{,}51}\cdot 10^{-6}+1}\right)^2}$$

$$= c\sqrt{1-\left(\frac{1}{1+0{,}000588}\right)^2} = c\sqrt{1-\frac{1}{1{,}001176}} = c\sqrt{1-0{,}998825}$$

$= 0{,}0342c$

б) $$v = c\sqrt{1-\frac{1}{\left(\frac{300}{0{,}511}+1\right)^2}} = c\cdot\sqrt{1-\frac{1}{(587+1)^2}} = c\sqrt{1-0{,}000\ldots}$$

$= 0{,}999999855\,c$

b) = $v = c\sqrt{1 - \frac{1}{\left(\frac{680}{938{,}28}+1\right)^2}} = c\sqrt{1 - \frac{1}{(1{,}7247)^2}} =$

$= c\sqrt{1 - 0{,}3362} = 0{,}8148\ c$

c) $v = c\sqrt{1 - \frac{1}{\left(\frac{10000}{938{,}28}+1\right)^2}} = c\sqrt{1 - \frac{1}{135{,}9}} =$

$= c\sqrt{0{,}99264} = 0{,}9963\ c$

Доза № 10

№ 702

Заряженная част. e, m проходит через $\varphi = k(x^2 - y^2)$

$k = const > 0$ (линза с сильной фокусировкой)

$t = 0$: (x_0, y_0, z_0); $v_0 = (0; 0; v_{0z})$. Определить движение част.

ур-е движения: $\frac{d\vec{p}}{dt} = e\vec{E}$

$\vec{E} = -\operatorname{grad}\varphi = -\nabla k(x^2 - y^2) = -k(2x; -2y; 0) = (-2kx; 2ky; 0)$

$$\begin{cases} \frac{dp_x}{dt} = -2kex \\ \frac{dp_y}{dt} = 2key \\ \frac{dp_z}{dt} = 0 \end{cases}$$

част. не рел.

$$\begin{cases} m \frac{d^2x}{dt^2} = -2kex \\ m \frac{d^2y}{dt^2} = 2key \\ p_z = m v_{0z} \Rightarrow z = E \cdot t + F \end{cases}$$

$$\begin{matrix} \ddot{x} + \frac{2ke}{m} x = 0 \\ \ddot{y} \quad \frac{2ke}{m} y = 0 \end{matrix} \Rightarrow \begin{matrix} x = A \cdot \cos\omega t + B \cdot \sin\omega t \\ y = C \cdot \operatorname{sh}\omega t + D \cdot \operatorname{ch}\omega t \end{matrix}$$

Нач. усл. $x(0) = x_0 = A$ $\quad y(0) = y_0 = D$ $\quad z(0) = z_0 = F$

$$\dot{x}(0) = -A\omega \sin\omega t + B\cdot\omega\cdot\cos\omega t \Big|_{t=0} = B\omega = 0 \Rightarrow B = 0$$

$$\dot{y}(0) = \omega C\cdot \mathrm{ch}\,\omega t + \omega D\cdot \mathrm{sh}\,\omega t \Big|_{t=0} = \omega C = 0 \Rightarrow C = 0$$

$$\dot{z}(0) = E = v_{0z}$$

$$\begin{cases} x = \underline{x_0 \cos\omega t} \quad \text{фокус} & x \le x_0 \,! \\ y = y_0\, \mathrm{ch}\,\omega t & ,\ \omega^2 = \dfrac{2ke}{m} \\ z = v_{0z} t + z_0 \end{cases}$$

№ 697

Релят. част. Однор. пост. внеш. поля $\vec{E} \parallel \vec{H} \parallel \vec{e}_z$

$t=0$: $x_0 = y_0 = z_0 = 0$; $\vec{p}_0 = (p_{0x}, 0, p_{0z})$

$x(\tau), y(\tau), z(\tau), t(\tau)$ - ?

Ур. е дв. я частицы в 4-мерн. виде:

$$\frac{dp^i}{ds} = mc \cdot \frac{du^i}{ds} = \frac{e}{c} F^{ik} u_k$$

$$ds = c\sqrt{1-\frac{v^2}{c^2}}\,dt \Rightarrow S = c\int_0^t \sqrt{1-\frac{v^2}{c^2}}\,dt \qquad t=0: S_0 = 0$$

$$F^{ik} = -F^{ki}; \quad F^{0\alpha} = -E^\alpha; \quad F^{\alpha\beta} = -e^{\alpha\beta\gamma} H^\gamma$$

$$F^{ik} = \begin{pmatrix} 0 & 0 & 0 & -E \\ 0 & 0 & -H & 0 \\ 0 & H & 0 & 0 \\ E & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} \frac{du^0}{ds} = \frac{e}{mc^2}(-E u_3) = \frac{eE}{mc^2} u^3 & (1) \\ \frac{du^1}{ds} = \frac{e}{mc^2}(-H \cdot u_2) = \frac{eH}{mc^2} u^2 & (2) \\ \frac{du^2}{ds} = \frac{e}{mc^2} H u_1 = -\frac{eH}{mc^2} u^1 & (3) \\ \frac{du^3}{ds} = \frac{e}{mc^2} E u_0 = \frac{eE}{mc^2} u^0 & (4) \end{cases}$$

μ (1) продиф. 1-ое ур-е по s

$$\frac{d^2u^0}{ds^2} = \frac{eE}{mc^2}\underbrace{\frac{du^3}{ds}} = \frac{eE}{mc^2}\underbrace{\frac{eE}{mc^2}u^0}$$

$$u^0 = A\, sh\left(\frac{eE}{mc^2}s\right) + B\, ch\left(\frac{eE}{mc^2}s\right)$$

Из нач. усл.

$t=0 \Rightarrow s=0 \quad u^0(t=0) = B \quad u^0 = \frac{p^0}{mc} = \frac{\varepsilon_0}{mc^2} \Rightarrow$

$$B = \frac{\varepsilon_0}{mc^2}$$

ℓ (4) Подставим u^0 в 4-ое ур-е, интегрируем:

$$u^3 = \cancel{\frac{eE}{mc^2}}\cancel{\frac{mc^2}{eE}}\left(A\, ch\left(\frac{eE}{mc^2}s\right) + \frac{\varepsilon_0}{mc^2}sh\left(\frac{eE}{mc^2}s\right)\right)$$

$$u^3(t=0) = A = \frac{p^3(t=0)}{mc} = \frac{p_{0z}}{mc}$$

$$\begin{cases} u^3 = \frac{p_{0z}}{mc}ch\left(\frac{eE}{mc^2}s\right) + \frac{\varepsilon_0}{mc^2}sh\left(\frac{eE}{mc^2}s\right) \\ u^0 = \frac{p_{0z}}{mc}sh\left(\frac{eE}{mc^2}s\right) + \frac{\varepsilon_0}{mc^2}ch\left(\frac{eE}{mc^2}s\right) \end{cases}$$

Аналогично из (3) и (2) ур-я получим:

$$\frac{d^2u^1}{ds^2} = -\left(\frac{eH}{mc^2}\right)^2 u^1 \Rightarrow u^1 = A\cos\left(\frac{eH}{mc^2}s\right) + \overset{=0}{B}\sin\left(\frac{eH}{mc^2}s\right) \to$$

подставим в (3),

$$u^1(t=0) = A = \frac{p^1(t=0)}{mc} = \frac{p_{0x}}{mc}$$

$$u^2 = \frac{eH}{\cancel{mc^2}} \cdot \frac{\cancel{mc^2}}{\cancel{eH}}\left(-A\sin\frac{eH}{mc^2}s + \overset{=0}{B}\cdot\cos\frac{eH}{mc^2}s\right)$$

$$u^2(t=0) = B = \frac{p^2(t=0)}{mc} = 0 \Rightarrow$$

$$\begin{cases} u^1 = \dfrac{p_{0x}}{mc}\cdot\cos\left(\dfrac{eH}{mc^2}\cdot s\right) \\ u^2 = -\dfrac{p_{0x}}{mc}\cdot\sin\left(\dfrac{eH}{mc^2}\cdot s\right) \end{cases}$$

$$u^i = \frac{dx^i}{ds}$$

Интегр. u^0 по ds

$$i=0 \quad u^0 = \frac{dx^0}{ds} = c\,\frac{dt}{ds} \qquad ct = \frac{p_{0z}}{\cancel{mc}}\cdot\frac{mc^{\cancel{2}}}{eE}\,ch\left(\frac{eE}{mc^2}s\right) +$$

$$+\frac{\varepsilon_0}{eE}\,sh\left(\frac{eE}{mc^2}s\right) + A$$

$$t=0 \quad \frac{p_{0z}}{eE}c + A = 0 \qquad ct = \frac{p_{0z}}{eE}\cdot c\left[ch\left(\frac{eE}{mc^2}s\right) - 1\right] +$$

$$+\frac{\varepsilon_0}{eE}\cdot sh\left(\frac{eE}{mc^2}s\right)$$

$i=1$: $x = \frac{p_{0x}}{mc} \cdot \frac{mc^2}{eH} \sin\left(\frac{eH}{mc^2} s\right) + B$

$x(t=0) = B = 0 \Rightarrow x = \frac{p_{0x} \cdot c}{eH} \cdot \sin\left(\frac{eH}{mc^2} s\right)$

$i=2$: $y = \frac{p_{0x} \cdot c}{eH} \cdot \cos\left(\frac{eH}{mc^2} s\right) + C \Rightarrow y(t=0) = 0 = \frac{p_{0x} \cdot c}{eH} + C$

$y = \frac{p_{0x} \cdot c}{eH} \left[\cos\left(\frac{eH}{mc^2} s\right) - 1\right]$

$i=3$: $z = \frac{p_{0z} \cdot c}{eE} \, sh\left(\frac{eE}{mc^2} \cdot s\right) + \frac{\mathcal{E}_0}{eE} \, ch\left(\frac{eE}{mc^2} \cdot s\right) +$

$+ \frac{\mathcal{E}_0}{eE} \left[ch\left(\frac{eE}{mc^2} s\right) - 1\right]$

$\tau = \frac{s}{c}$ – собств. время

$x = \frac{p_{0x} \cdot c}{eH} \cdot \sin\left(\frac{eH}{mc} \cdot \tau\right)$

$y = \frac{p_{0x} \cdot c}{eH} \left[\cos\left(\frac{eH}{mc} \cdot \tau\right) - 1\right]$

$z = \frac{p_{0z} \cdot c}{eE} \, sh\left(\frac{eE}{mc} \cdot \tau\right) + \frac{\mathcal{E}_0}{eE} \left[ch\left(\frac{eE}{mc} \tau\right) - 1\right]$

$ct = \frac{p_{0z} c}{eE} \left[ch\left(\frac{eE}{mc} \tau\right) - 1\right] + \frac{\mathcal{E}_0}{eE} \cdot sh\left(\frac{eE}{mc} \cdot \tau\right)$

№ 61

$\dot{\vec{v}}(\vec{v}, \vec{E}, \vec{H})$ - ?

Ур-е дв-я заряда в э.м.п. (3-мерном вид):

$$\frac{d\vec{p}}{dt} = e\cdot\vec{E} + \frac{e}{c}[\vec{v}\times\vec{H}]$$

$$\vec{p} = \frac{\mathcal{E}_k \cdot \vec{v}}{c^2} \qquad \frac{d\mathcal{E}_k}{dt} = e\vec{E}\cdot\vec{v} \qquad \vec{v} = \frac{c^2\vec{p}}{\mathcal{E}_k}$$

$$\frac{d\vec{p}}{dt} = \frac{\mathcal{E}_k}{c^2}\dot{\vec{v}} + \frac{\vec{v}}{c^2}\cdot e\vec{E}\vec{v} = e\cdot\vec{E} + \frac{e}{c}[\vec{v}\times\vec{H}]$$

$$\mathcal{E}_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \dot{\vec{v}} = \frac{1}{m}\sqrt{1-\frac{v^2}{c^2}}\left(e\vec{E} + \frac{e}{c}[\vec{v}\times\vec{H}] - \frac{e\vec{v}}{c^2}(\vec{E}\cdot\vec{v})\right) =$$

$$= \frac{e}{m}\sqrt{1-\frac{v^2}{c^2}}\left(\vec{E} + \frac{1}{c}[\vec{v}\times\vec{H}] - \frac{1}{c^2}\vec{v}(\vec{E}\cdot\vec{v})\right)$$

№ 62

ур-е пространственного осциллятора в поле

$$\begin{cases} \ddot{x} + \omega_0^2 x = \frac{1}{m}\frac{dp_x}{dt} \\ \ddot{y} + \omega_0^2 y = \frac{1}{m}\frac{dp_y}{dt} \\ \ddot{z} + \omega_0^2 z = \frac{1}{m}\frac{dp_z}{dt} \end{cases}$$

$\vec{H} = (0; 0; H)$; $\vec{E} = 0$ $\qquad \frac{d\vec{p}}{dt} = \frac{e}{c}[\vec{v} \times \vec{H}]$

$$\frac{d\vec{p}}{dt} = \frac{e}{c}\begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \dot{x} & \dot{y} & \dot{z} \\ 0 & 0 & H \end{vmatrix}$$

$$\frac{dp_x}{dt} = \frac{e}{c}H\dot{y}$$

$$\frac{dp_y}{dt} = -\frac{eH}{c}\cdot\dot{x}$$

$$\frac{dp_z}{dt} = 0$$

$$\begin{cases} \ddot{x} + \omega_0^2 x = \frac{eH}{mc}\dot{y} \\ \ddot{y} + \omega_0^2 y = -\frac{eH}{mc}\cdot\dot{x} \quad \Big| \cdot i \\ \ddot{z} + \omega_0^2 z = 0 \end{cases} \quad +$$

$$(\ddot{x} + i\ddot{y}) + \omega_0^2(x + iy) = \frac{eH}{mc}(\dot{y} - i\dot{x}) = -\frac{ieH}{mc}(\dot{x} + i\dot{y})$$

$x + iy = \xi$

$\ddot{\xi} + \omega_0^2 \xi = -\frac{ieH}{mc}\dot{\xi}$ $\qquad$ $\ddot{\xi} + \frac{ieH}{mc}\dot{\xi} + \omega_0^2\xi = 0$

решение $\xi = e^{\pm i\omega t}$

$-\omega^2 - \frac{eH}{mc}\omega + \omega_0^2 = 0$ $\qquad$ $-\omega^2 + \frac{eH}{mc}\omega + \omega_0^2 = 0$

$\Downarrow$ $\qquad$ $\Downarrow$

$\omega = -\frac{eH}{2mc} \pm \sqrt{\omega_0^2 + \frac{1}{4}\left(\frac{eH}{mc}\right)^2}$ $\qquad$ $\omega = \frac{eH}{2mc} \pm \sqrt{\omega_0^2 + \frac{1}{4}\left(\frac{eH}{mc}\right)^2}$

при $H = 0$ $\quad \omega = \omega_0$

$$\omega = \sqrt{\omega_0^2 + \frac{1}{4}\left(\frac{eH}{mc}\right)^2} \pm \frac{eH}{2mc}$$

№ 703

$$L = -mc^2\sqrt{1-\frac{v^2}{c^2}} + \frac{e}{c}\vec{v}\vec{A} - e\varphi$$

$$d\vec{l} = \vec{e_r}\,dr + \vec{e_\alpha}\, r\, d\alpha + \vec{e_z}\,dz$$

$$\vec{v} = \vec{e_r}\cdot\dot{r} + \vec{e_\alpha}\cdot r\dot{\alpha} + \vec{e_z}\cdot\dot{z} = \vec{e_r}\cdot v_r + \vec{e_\alpha}\cdot v_\alpha + \vec{e_z}\, v_z$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial q} \qquad \boxed{L = -mc^2\sqrt{1-\frac{1}{c^2}(\dot{r}^2+r^2\dot{\alpha}^2+\dot{z}^2)} + \frac{e}{c}(\dot{r}A_r + r\dot{\alpha}A_\alpha + \dot{z}A_z) - e\varphi}$$

$q_1 = r \qquad q_2 = \alpha \qquad q_3 = z$

$$\frac{d}{dt}\left(\frac{m\dot{r}}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{mr\cdot\dot{\alpha}^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{e}{c}(\vec{v}\times\vec{H})_r + e\cdot E_r$$

1) $q = \alpha$

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\alpha}} = \frac{d}{dt}\left(\frac{\partial}{\partial\dot{\alpha}}\left(-mc^2\sqrt{1-\frac{1}{c^2}(\dot{r}^2+r^2\dot{\alpha}^2+\dot{z}^2)}\right.\right. +$$

$$+\frac{e}{c}(\dot{r}\cdot A_r + r\dot{\alpha}A_\alpha + \dot{z}\cdot A_z) - e\varphi\Big) =$$

$$= \frac{d}{dt}\left(\frac{m\cancel{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{r^2\cdot\dot{\alpha}}{\cancel{c^2}} + \frac{e}{c}r\cdot A_\alpha\right) =$$

$$= \frac{d}{dt}\left(\frac{mr^2\cdot\dot{\alpha}}{\sqrt{1-\frac{v^2}{c^2}}}\right) + \frac{e}{c}\dot{r}A_\alpha + \frac{e}{c}r\frac{dA_\alpha}{dt} =$$

$$= \left\{ \frac{dA_\alpha}{dt} = \frac{\partial A_\alpha}{\partial t} + \frac{\partial A_\alpha}{\partial r}\frac{dr}{dt} + \frac{\partial A_\alpha}{\partial\alpha}\frac{d\alpha}{dt} + \frac{\partial A_\alpha}{\partial z}\frac{dz}{dt} = \right.$$

$$= \frac{\partial A_\alpha}{\partial t} + (\vec{v}\nabla)A_\alpha\Big\} =$$

$+\overline{e_z}\, v_z$

$r\dot\alpha A_\alpha + \dot z A_z) -$

$- e\psi$

$$= \frac{d}{dt}\left(\frac{mr^2\dot\alpha}{\sqrt{1-\frac{v^2}{c^2}}}\right) + \frac{e}{c}\dot r A_\alpha + \frac{e}{c} r\left(\frac{\partial A_\alpha}{\partial t} - v_r\frac{\partial A_\alpha}{\partial r} + \frac{v_\alpha}{r}\frac{\partial A_\alpha}{\partial \alpha} + \right.$$

$$\left. + v_z\frac{\partial A_\alpha}{\partial z}\right)$$

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial \alpha} = \frac{e}{c}\left(\dot r\frac{\partial A_r}{\partial \alpha} r + r\dot\alpha\frac{\partial A_\alpha}{\partial \alpha} + \dot z\frac{\partial A_z}{\partial \alpha}\right) - e\frac{\partial \psi}{\partial \alpha}$$

$$\frac{d}{dt}\left(\frac{mr^2\dot\alpha}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{e}{c}\left(\dot r\frac{\partial A_r}{\partial \alpha} + r\dot\alpha\frac{\partial A_\alpha}{\partial \alpha} + \dot z\frac{\partial A_z}{\partial \alpha}\right) - e\frac{\partial \psi}{\partial \alpha} -$$

$$- \frac{e}{c}\dot r A_\alpha - \frac{e}{c} r\left(\frac{\partial A_\alpha}{\partial t} + v_r\frac{\partial A_\alpha}{\partial r} + \frac{v_\alpha}{r}\frac{\partial A_\alpha}{\partial \alpha} + v_z\frac{\partial A_\alpha}{\partial z}\right) =$$

$$(\operatorname{rot}\vec A)_r = \frac{1}{r}\frac{\partial}{\partial r}(rA_\alpha) - \frac{1}{r}\frac{\partial A_r}{\partial \alpha}$$

$$(\operatorname{rot}\vec A)_\alpha = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \qquad \vec E = -\frac{1}{c}\frac{\partial \vec A}{\partial t} - \operatorname{grad}\psi$$

$$(\operatorname{rot}\vec A)_z = \frac{1}{r}\frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z}$$

$$\nabla = \overline{e_r}\frac{\partial}{\partial r} + \overline{e_\alpha}\frac{1}{r}\frac{\partial}{\partial \alpha} + \overline{e_z}\frac{\partial}{\partial z} \qquad E_\alpha = -\frac{1}{c}\frac{\partial A_\alpha}{\partial t} - \frac{1}{r}\frac{\partial \psi}{\partial \alpha}$$

$$= er\left(-\frac{1}{r}\frac{\partial \psi}{\partial \alpha} - \frac{1}{c}\frac{\partial A_\alpha}{\partial t}\right) + \frac{er}{c}\left(\frac{v_r}{r}\frac{\partial A_r}{\partial \alpha} + \frac{v_z}{r}\frac{\partial A_z}{\partial \alpha} - \frac{\dot r}{r}A_\alpha - \right.$$

$$\left. - v_r\frac{\partial A_\alpha}{\partial z} - v_z\frac{\partial A_\alpha}{\partial r}\right) = erE_\alpha + \frac{er}{c}\left[v_r\left(\frac{1}{r}\frac{\partial A_r}{\partial \alpha} - \frac{\partial A_\alpha}{\partial r}\right) - \frac{A_\alpha}{r}\right) +$$

$$+ v_z\left(\frac{1}{r}\frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z}\right)\Big) = erE_\alpha + \frac{er}{c}(v_z H_r - v_r H_r) = er\left(E_\alpha + \frac{1}{c}(\vec v \times \vec H)_\alpha\right)$$

$$\frac{d}{dt}\left(\frac{mr^2\dot\alpha}{\sqrt{1-\frac{v^2}{c^2}}}\right) = er\left(E_\alpha + \frac{1}{c}(\vec v \times \vec H)_\alpha\right)$$

3) $q = z$

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot z}\left(-mc^2\sqrt{1-\frac{1}{c^2}\left(\dot r^2 + r^2\dot\alpha^2 + \dot z^2\right)} + \frac{e}{c}\left(\dot r A_r + r\dot\alpha A_\alpha + \dot z A_z\right)\right.\right.$$

$$\left.\left. - e\varphi\right)\right) = \frac{d}{dt}\left(\frac{mc^2\dot z}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{e}{c}A_r\right) = \frac{d}{dt}\left(\frac{m\dot z}{\sqrt{1-\frac{v^2}{c^2}}}\right) + \frac{e}{c}\frac{dA_z}{dt} =$$

$$= \frac{d}{dt}\left(\frac{m\dot z}{\sqrt{1-\frac{v^2}{c^2}}}\right) + \frac{e}{c}\frac{\partial A_z}{\partial t} + \frac{e}{c}\left(v_r \frac{\partial A_z}{\partial r} + \frac{v_\alpha}{r}\frac{\partial A_z}{\partial \alpha} + v_z\frac{\partial A_z}{\partial z}\right)$$

$$\frac{\partial L}{\partial z} = \frac{e}{c}\left(\dot r\frac{\partial A_r}{\partial z} + r\dot\alpha\frac{\partial A_\alpha}{\partial z} + \dot z\frac{\partial A_z}{\partial z}\right) - e\frac{\partial \varphi}{\partial z}$$

$$\frac{d}{dt}\left(\frac{m\dot z}{\sqrt{1-\frac{v^2}{c^2}}}\right) = -e\frac{\partial\varphi}{\partial z} - \frac{e}{c}\frac{\partial A_z}{\partial t} + \frac{e}{c}\left(v_r\frac{\partial A_r}{\partial z} + v_\alpha\frac{\partial A_\alpha}{\partial z}\right.$$

$$\left. - v_z\frac{\partial A_z}{\partial r} - \frac{v_\alpha}{r}\frac{\partial A_z}{\partial \alpha}\right) = eE_z + \frac{e}{c}\left(v_r H_\alpha - v_\alpha H_r\right) =$$

$$= eE_z + \frac{e}{c}(\vec v \times \vec H)_z$$

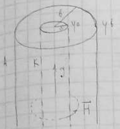
$$\frac{d}{dt}\left(\frac{m\dot z}{\sqrt{1-\frac{v^2}{c^2}}}\right) = eE_z + \frac{e}{c}(\vec v \times \vec H)_z$$

$$\frac{d}{dt}\left(\frac{m\dot r}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{mr\dot\alpha^2}{\sqrt{1-\frac{v^2}{c^2}}} + eE_r + \frac{e}{c}(\vec v \times \vec H)_r$$

$$\frac{d}{dt}\left(\frac{mr^2\dot\alpha}{\sqrt{1-\frac{v^2}{c^2}}}\right) = er\left(E_\alpha + \frac{1}{c}(\vec v \times \vec H)_\alpha\right)$$

$$\frac{d}{dt}\left(\frac{m\dot z}{\sqrt{1-\frac{v^2}{c^2}}}\right) = eE_z + \frac{e}{c}(\vec v \times \vec H)_z$$

№ 705.



Анод испускает $\bar{e}$ с $v = 0$

по аноду течёт распределённый ток J

это $V_{кр}$

$|\psi_b - \psi_a|_{min} = V_{min}$, чтобы $\bar{e}$ долетели катода

p_α, p_z

$\mathcal{E} = const$

$$\mathcal{E} = \sqrt{c^2p^2 + m^2c^4} - e\psi \qquad \mathcal{E} > 0$$

$$\mathcal{E}(r=a) = mc^2 - e\psi_a \qquad \mathcal{E}(r=b) = \sqrt{m^2c^4 + c^2 p_z^{(1)2}} - e\psi_b$$

$$mc^2 + e(\psi_b - \psi_a) = \sqrt{m^2c^4 + c^2 p_z^{(1)2}}$$

Для того, чтобы $\bar{e}$ ускор. $\vec{E}$ полем, оно д. б. напр. к оси z, т.е. $\psi_b - \psi_a = V > 0 \Rightarrow$

$$e \cdot V_{кр} = \sqrt{m^2c^4 + c^2 p_z^{(1)2}} - mc^2 \Rightarrow$$

$$V_{кр} = \sqrt{\frac{m^2c^4}{e^2} + \frac{c^2}{e^2} p_z^{(1)2}} - \frac{mc^2}{e} \qquad p_z^{(1)} - ? \text{ из практики:}$$

$$\frac{dp_z}{dt} = F_z = eE_z - \frac{e}{c}(\vec{v} \times \vec{H})_z = \left\{ \vec{E} = (E, 0, 0);\ \vec{H} = (0, H, 0) \right\} =$$

$$= -\frac{e}{c} \vec{v}_r \cdot H = \Big/ H = \frac{2J}{cr} \Big/ = -\frac{e}{c}\frac{2J}{c}\frac{dr}{r \cdot dt}$$

интегр. от a до b, с учетом $p_z^{(0)} = 0$

$$p_z^{(1)} = -\frac{2J\cdot e}{c^2}\ln r\Big|_a^b = -\frac{2J\cdot e}{c^2}\cdot\ln\frac{b}{a} \Rightarrow$$

$$\Rightarrow\quad V_{\text{кр.}} = \sqrt{\frac{c^2}{e^2}\cdot\frac{4J^2e^2}{c^4\,c^2}\ln^2\frac{b}{a} + \frac{m^2c^4}{e^2}} - \frac{mc^2}{e}$$

$$V_{\text{кр}} = \sqrt{\frac{4J^2}{c^2}\ln^2\frac{b}{a} + \frac{m^2c^4}{e^2}} - \frac{mc^2}{e}$$

№ 605

S: $\vec{E} \perp \vec{H}$ а) $H^2 - E^2 = H'^2 - E'^2 = -E'^2 \Rightarrow$

S': а) $H' = 0$ $E > H$

б) $E' = 0$

S' движ. относ. S с V Пусть $\vec{V} \perp (\vec{E}, \vec{H})$

$\vec{V}$ - ?

$$\vec{H}' = 0 = \frac{\vec{H} - \frac{1}{c}\vec{V}\times\vec{E} - \frac{1}{V^2}\left(1 - \sqrt{1 - \frac{V^2}{c^2}}\right)\cancel{\vec{V}(\vec{V}\vec{H})}}{\sqrt{1 - \frac{V^2}{c^2}}}$$

$$\vec{H} = \frac{1}{c}\vec{V}\times\vec{E} \quad \Big| \times\vec{E}$$

$$\vec{E}\times\vec{H} = \frac{1}{c}\vec{E}\times(\vec{V}\times\vec{E}) = \frac{1}{c}\vec{V}E^2 - \frac{1}{c}\vec{E}\cancel{(\vec{E}\vec{V})} \Rightarrow$$

$$\vec{V} = c\,\frac{\vec{E}\times\vec{H}}{E^2}$$

введем сист. S'', кот. движ. относ. S' со скор. V':

$$\vec{H}'' = \frac{\cancel{\vec{H}'} + \frac{1}{c}\vec{V}'\times\vec{E}' - \frac{1}{V'^2}\left(1 - \sqrt{1 - \frac{V'^2}{c^2}}\right)\cancel{\vec{V}'(\vec{V}'\vec{H}')}}{\sqrt{1 - \frac{V'^2}{c^2}}}$$

требуем, чтобы $\vec{H}'' = 0 \Rightarrow \frac{1}{c}\vec{V}'\times\vec{E}' = 0 \Rightarrow$

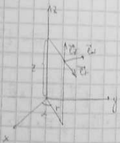
в $\forall$ сист., движущейся вдоль $\vec{E}'$ с произвольной скор. магн. поле также будет отсутствовать б) $H^2 - E^2 = H'^2 - E'^2 = H'^2 \Rightarrow H > E$

$$\vec{E}' = 0 = \frac{\vec{E} + \frac{1}{c}\vec{V}\times\vec{H} - \cancel{\frac{1}{V^2}\left(1-\sqrt{1-\frac{V^2}{c^2}}\right)\vec{V}(\vec{V}\vec{E})}}{\sqrt{1-\frac{V^2}{c^2}}} \Rightarrow$$

$$\Rightarrow \vec{E} = \frac{1}{c}(\vec{H}\times\vec{V}) \Big| \times \vec{H} \Rightarrow \vec{H}\times\vec{E} = \frac{1}{c}\vec{H}\times(\vec{H}\times\vec{V}) =$$

$$= \cancel{\frac{1}{c}\vec{H}(\vec{H}\vec{V})} - \frac{1}{c}\vec{V}\cdot H^2$$

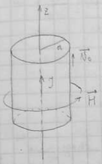
$$\vec{V} = c\,\frac{\vec{E}\times\vec{H}}{H^2}$$



№706

Цилиндр. провод рад. a, течёт ток J. $\vec{v}_0$ – нач. скор. $\bar{e}$, который вылетает с провода, $\vec{v}_0 \parallel \vec{e}_z$

b – ? (max расст. на кот. удалится $\bar{e}$ от оси цил.)



протекание тока сопровожд. появл. магн. поля $\vec{H}$ вокруг провода.

($\vec{v}_0 \uparrow\uparrow \vec{e}_z$, иначе магн. составл. силы Лоренца отклоняла бы $\bar{e}$ к z)

$\bar{e}$ опишет дугу (поворот $\bar{e}$ будет $\downarrow$ с $\uparrow$ расст. до z), т.к. $H \sim \frac{1}{r}$

b будет достигнуто, когда $\vec{v} \uparrow\downarrow \vec{e}_z$

$$\mathcal{E} = \sqrt{c^2p^2 + m^2c^4} = const \qquad p_\alpha^{(b)} = p_\alpha^{(a)} = p_r^{(b)} = p_r^{(a)} = 0$$

$$\mathcal{E}(r=a) = \sqrt{c^2(p_z^{(a)})^2 + m^2c^4}$$

$$\mathcal{E}(r=b) = \sqrt{c^2(p_z^{(b)})^2 + m^2c^4} \Rightarrow (p_z^{(a)})^2 = (p_z^{(b)})^2 \Rightarrow$$

$$|p_z^{(a)}| = |p_z^{(b)}|$$

из ур-я: $\frac{dp_z}{dt} = F_z = -\frac{e}{c}(\vec{v} \times \vec{H})_z$ получим:

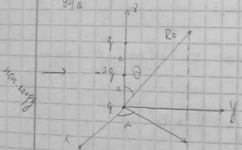
$$p_z^{(1)} - p_z^{(2)} = -\frac{2Je}{c^2} \ln\frac{b}{a}$$

$$p_0 = \frac{mv_0}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (p_z^{(2)} > 0, \quad p_z^{(1)} < 0)$$

$$2p_z^{(2)} = 2p_0 = \frac{2Je}{c^2} \ln\frac{b}{a} \Rightarrow \frac{b}{a} = \exp\left\{\frac{p_0 c^2}{Je}\right\} \Rightarrow$$

$$b = a \cdot \exp\left\{\frac{p_0 \cdot c^2}{J \cdot e}\right\}$$

94 а



Линейный квадруполь

$\varphi(R_0)$? $\quad R_0 \to \infty$

$$\varphi = \frac{\sum e}{R_0} + \frac{\vec{n}\cdot\vec{d}}{R_0^3} + \frac{D_{\alpha\beta} n_\alpha n_\beta}{2R_0^3} \qquad \vec{n} = \frac{\vec{R_0}}{R_0}$$

при $R_0 \to \infty$ не учитыв. 1-е слагаемое

$$\left.\begin{aligned} d_x &= \sum e \cdot z = q \cdot 0 - 2q \cdot a + q \cdot 2a = 0 \\ d_y &= \sum e y = 0 \\ d_z &= \sum e x = 0 \end{aligned}\right\} \vec{d} = 0$$

2 с шп. тоже уходит

$$\varphi = \frac{D_{\alpha\beta}\, n_\alpha n_\beta}{2R_0^3}$$, $D_{\alpha\beta}$ - тензор квадрупольного момента

If шп. зарядов имеет ось симметрии выше 2-го порядка, то $D_{xx}+D_{yy}+D_{zz}=0$. $D_{xx}=D_{yy}$ (симм. пор. k) $\varphi \neq \frac{2\pi}{k}$

$$D_{xx} = D_{yy} = -\frac{1}{2} D_{zz}; \quad D_{\alpha\beta} = 0$$

по опред. $D_{\alpha\beta} = \sum e(3x_\alpha x_\beta - \delta_{\alpha\beta}\cdot r^2) \Rightarrow$

$$D_{zz} = -2q\cdot(3a^2 - a^2) + q\cdot(12a^2 - 4a^2) = -4q\cdot a^2 + 8q\cdot a^2 = 4q\cdot a^2$$

$$D_{xx} = D_{yy} = -2q\cdot a^2 \Rightarrow$$

$$\varphi = \frac{D_{zz}}{2R_0^3}\left(n_z\cdot n_z - \frac{n_y\cdot n_y}{2} - \frac{n_x\cdot n_x}{2}\right) = \frac{2q\,a^2}{R_0^3}\Big(\cos^2\theta -$$

$$- \frac{\sin^2\theta\cdot\sin^2\alpha}{2} - \frac{\sin^2\theta\cdot\cos^2\alpha}{2}\Big) = \frac{2q\cdot a^2}{R_0^3}\left(\cos^2\theta - \frac{\sin^2\theta}{2}(\sin^2\alpha + \cos^2\alpha)\right) =$$

$$= \frac{2q\,a^2}{R_0^3}\left(\cos^2\theta - \frac{\sin^2\theta}{2}\right) = \frac{2q\,a^2}{R_0^3}\,\frac{1}{2}\left(3\cos^2\theta - 1\right)$$

полиномы Лежандра. $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left((x^2-1)^n\right)$

явный вид полин. Лежандра 2 порядка:

$$P_2(x) = \frac{1}{8}\frac{d^2}{dx^2}\left(x^4 - 2x^2 + 1\right) = \frac{1}{8}\left(12x^2 - 4\right) = \underline{\underline{\frac{1}{2}(3x^2-1)}}$$

$x \to \cos\theta$

$$P_2(\cos\theta) = \frac{1}{2}\left(3\cos^2\theta - \sin^2\theta - \cos^2\theta\right) =$$

$$= \frac{1}{2}\left(2\cos^2\theta - \sin^2\theta\right) = \cos^2\theta - \frac{\sin^2\theta}{2}$$

$$\underset{R_0\to\infty}{V(R_0)} = \frac{2q\cdot a^2}{R_0^3}\left(\cos^2\theta - \frac{\sin^2\theta}{2}\right) = \frac{2q\cdot a^2}{R_0^3}\, P_2(\cos\theta)$$

№ 119

Б сфер. коорд. $\varphi(r) = q\,\dfrac{e^{-\alpha r}}{r}$ $\quad \alpha, q$: const

$\rho(r)$ - ?

$\Delta\varphi = -4\pi\rho \Rightarrow \rho = -\dfrac{1}{4\pi}\cdot\Delta\varphi$

$\Delta f(r) = \dfrac{1}{r}\cdot\dfrac{d^2}{dr^2}\{r\,f(r)\}$ имеем $\rho = -\dfrac{1}{4\pi}\Delta\left[\dfrac{q e^{-\alpha r}}{r}\right] =$

$= -\dfrac{q}{4\pi}\Delta\left[\dfrac{e^{-\alpha r}-1}{r} + \dfrac{1}{r}\right] = -\dfrac{q}{4\pi}\Delta\left[\dfrac{e^{-\alpha r}-1}{r}\right] - \dfrac{q}{4\pi}\Delta\left(\dfrac{1}{r}\right) =$

$= \left\{\Delta\dfrac{1}{r} = -4\pi\delta(\vec{r})\right\} = -\dfrac{q}{4\pi r}\cdot\dfrac{d^2}{dr^2}\left[e^{-\alpha r}-1\right] + q\cdot\delta(\vec{r}) =$

$= -\dfrac{q}{4\pi r}\left[\alpha^2\cdot e^{-\alpha r}\right] + q\cdot\delta(\vec{r})$

№ 83

Б ат водорода: $\rho(r) = -\frac{e_0}{\pi a^3}\exp\left[-\frac{2r}{a}\right]$

$a = const$ (боровский рад. ат)

e_0

$\varphi - ?$ $E - ?$

из зад № 81

$$\int_0^r e^{\gamma r'} r'^2 dr' = \frac{d^2}{d\gamma^2}\int_0^r e^{\gamma r'} dr'$$

1) $E_{0r} = \frac{4\pi}{r^2}\int_0^r \rho(r') r'^2 dr' = -\frac{4\pi}{r^2}\int_0^r \frac{e_0}{\pi a^3} e^{-\frac{2r}{a}} r^2 dr =$

$= \frac{4e_0 a}{r^2 a^3 2}\int_0^r r^2 \cdot de^{-\frac{2r}{a}} = \frac{2e_0}{r^2 a^2} r^2 e^{-\frac{2r}{a}}\Big|_0^r - \frac{4e_0}{r^2 a^2}\int_0^r r\,e^{-\frac{2r}{a}}$

$= \frac{2e_0 e^{-\frac{2r}{a}}}{a^2} + \frac{4e_0}{r^2 a^2}\cdot\frac{a}{2}\int_0^r r\cdot de^{-\frac{2r}{a}} = \frac{2e_0}{ar}\cdot e^{-\frac{2r}{a}} + \frac{2e_0}{r^2 a} r e^{-\frac{2r}{a}}$

$-\frac{4e_0}{r^2 a^2}\cdot\frac{a}{2}\int_0^r e^{-\frac{2r}{a}} dr = \frac{2e_0}{a^2}\cdot e^{-\frac{2r}{a}} + \frac{2e_0}{2a}\cdot e^{-\frac{2r}{a}} + \frac{4e_0 a}{2r^2 a^2 2}\cdot$

$\cdot\left[e^{-\frac{2r}{a}} - 1\right] = \frac{2e_0}{a^2}\cdot e^{-\frac{2r}{a}} - \frac{e_0}{r^2}\left[1 - \frac{2r}{a}e^{-\frac{2r}{a}} - e^{-\frac{2r}{a}}\right] =$

$= \frac{2e_0}{a^2} e^{-\frac{2r}{a}} - \frac{e_0}{r^2}\left[1 - \left(\frac{2r}{a}+1\right)e^{-\frac{2r}{a}}\right] = E_{0r}$

2) $\varphi_e(r) = 4\pi \frac{1}{r}\int_0^r \rho(r')r'^2\, dr' + 4\pi \int_r^\infty \rho(r')r'\, dr'$;

$$-4\cancel{\pi}\int_r^\infty \frac{e_0}{\cancel{\pi} a^3} e^{-\frac{2r'}{a}} r'\, dr' = \frac{4e_0}{a^3}\cdot\frac{a}{2}\int_r^\infty r'\, de^{-\frac{2r'}{a}} =$$

$$= \frac{2e_0}{a^2} r' e^{-\frac{2r'}{a}}\Big|_r^\infty + \frac{2e_0}{a^2}\cdot\frac{a}{2}\int_r^\infty de^{-\frac{2r'}{a}} =$$

$$= -\frac{2e_0}{a^2} r\cdot e^{-\frac{2r}{a}} + \frac{e_0}{a} e^{-\frac{2r'}{a}}\Big|_r^\infty = -\frac{2e_0}{a^2}\cdot r\cdot e^{-\frac{2r}{a}} - \frac{e_0}{a} e^{-\frac{2r}{a}}$$

$$4\pi\cdot\frac{1}{r}\cdot\int_0^r \rho(r')r'^2\, dr' = r\cdot E_{er} = -\frac{e_0}{r}\left[1-\left(\frac{2r}{a}+1\right)e^{-\frac{2r}{a}}\right] + \frac{2e_0 r}{a^2} e^{-\frac{2r}{a}} \Rightarrow$$

$$\Rightarrow \varphi_e(r) = -\frac{e_0}{r} + \frac{2r}{a}\cdot\frac{e_0}{r}\cdot e^{-\frac{2r}{a}} - \frac{e_0}{r} e^{-\frac{2r}{a}} + \frac{2e_0 r}{a^2} e^{-\frac{2r}{a}} - \frac{2e_0}{a^2} r\cdot e^{-\frac{2r}{a}} -$$

$$- \frac{e_0}{a}\cdot e^{-\frac{2r}{a}} = -\frac{e_0}{r}\left[1-e^{-\frac{2r}{a}}\right] + \frac{e_0}{a} e^{-\frac{2r}{a}}$$

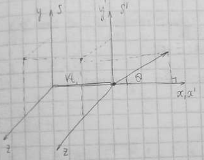
$$\varphi_e(r) = -\frac{e_0}{r}\left(1-e^{-\frac{2r}{a}}\right) + \frac{e_0}{a} e^{-\frac{2r}{a}}$$

3) $\varphi_p(r) = \frac{e_0}{r}$ $\quad E_{pr} = \frac{e_0}{r^2}$ $\quad \varphi(r) = -\frac{e_0}{r}\left(1-e^{-\frac{2r}{a}}\right) + \frac{e_0}{a} e^{-\frac{2r}{a}} + \frac{e_0}{r}$

$$E(r) = -\frac{e_0}{r^2}\left(1-\left(\frac{2r}{a}+1\right)e^{-\frac{2r}{a}}\right) + \frac{2e_0}{a^2} e^{-\frac{2r}{a}} + \frac{e_0}{r^2}$$

№ 611.

равномерно движ. заряд; показать, что E зар. „сплющивается" в напр. движ. (ослабление поля E на линии дв-я зар). Сравни с формулой $E_{\parallel} = E'_{\parallel}$



заряд наход. в нач. коорд. S', кот. дв. со скор. V относ лабор. сист. отн. S.

из № 610. $$\vec{E} = \frac{e\vec{R}\left(1-\frac{V^2}{c^2}\right)}{R^3\left(1-\frac{V^2}{c^2}\sin^2\theta\right)^{3/2}}$$

фиксир. R, V смотрим зав-ть $E(\theta)$. При $\theta = 0, \pi$ E - min, при $\theta = \frac{\pi}{2}; \frac{3\pi}{2}$ max

$$\theta = 0, \pi: \quad E_{\parallel} = \frac{e}{R_{\parallel}^2}\left(1-\frac{V^2}{c^2}\right) = \frac{e}{R^2}\left(1-\frac{V^2}{c^2}\right)$$

$$\theta = \frac{\pi}{2}; \frac{3\pi}{2}: \quad E_{\perp} = \frac{e}{R_{\perp}^2\sqrt{1-\frac{V^2}{c^2}}} = \frac{e}{R^2}\left(1-\frac{V^2}{c^2}\right)^{-\frac{1}{2}}$$

$R_x = R_x' \sqrt{1-\frac{V^2}{c^2}}$

$$E_x = \frac{e\left(1-\frac{V^2}{c^2}\right)}{R_0^2} = \frac{e\left(1-\frac{V^2}{c^2}\right)}{\left(R_x' \sqrt{1-\frac{V^2}{c^2}}\right)^2} = \frac{e}{R_x'^2} = E_{\|}'$$

№ 638 §

$\vec{E} \perp \vec{H}$, $\vec{E}$ и $\vec{H}$ - однор. пост. эл. и магн. поля

$\vec{H} = (0; 0; H)$ $\quad \vec{E} = (0; E; 0)$

$t = 0 \Rightarrow \vec{p}_0 \quad \vec{F}_L = 0$ З-н движения?

$$\frac{dp^i}{ds} = \frac{e}{c} F^{ik} u_k$$

$p^i = mcu^i$

$$\frac{du^i}{ds} = \frac{e}{mc^2} F^{ik} u_k \qquad F^{0\alpha} = -E^\alpha, \quad F^{\alpha\beta} = -e^{\alpha\beta\gamma} H^\gamma$$

$F^{ik} = -F^{ki}$

$$F^{ik} = \begin{pmatrix} 0 & 0 & -E & 0 \\ 0 & 0 & -H & 0 \\ E & H & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} \frac{du^0}{ds} = \frac{e}{mc^2} E u^2 & (1) \\ \frac{du^1}{ds} = \frac{e}{mc^2} H u^2 & (2) \\ \frac{du^2}{ds} = \frac{e}{mc^2} E u^0 - \frac{e}{mc^2} H u^1 & (3) \\ \frac{du^3}{ds} = 0 \Rightarrow u^3 = const & (4) \end{cases}$$

$p^i = mcu^i \Rightarrow$

$\Rightarrow x^3 = z = \frac{p_{0z}}{mc}$

Продифф. (3)

$\frac{d^2u^2}{ds^2} = \frac{e}{mc^2}$

$= \left(\frac{e}{mc^2}\right)^2 \cdot \{E^2 -$

$E > H$

$u^2 = A$

Из н.у.

$ds = c\,d$

$u^2(t=0) =$

Из (5)

Подст.

$p^2 = mcu^2 \Rightarrow p^3 = mcu^3 \quad mc\dfrac{dx^3}{ds} = p_{0z} \Rightarrow$

$\Rightarrow x^3 = z = \dfrac{p_{0z}}{mc}\, s$

Продиф. (3) по s

$$\frac{d^2u^2}{ds^2} = \frac{e}{mc^2}\left\{ E\cdot\frac{du^0}{ds} - H\cdot\frac{du^1}{ds} \right\} = \frac{e}{mc^2}\left\{ \frac{e}{mc^2}E^2u^2 - \frac{e}{mc^2}H^2u^2 \right\} =$$

$$= \left(\frac{e}{mc^2}\right)^2 \cdot \left\{ E^2 - H^2 \right\} u^2$$

$E > H$

$$u^2 = A\,\mathrm{ch}\left(\sqrt{E^2-H^2}\,\frac{e}{mc^2}\, s\right) + B\,\mathrm{sh}\left(\sqrt{E^2-H^2}\,\frac{e}{mc^2}\, s\right) \quad (5)$$

Из н.у. опр. A и B:

$$ds = c\,dt\sqrt{1-\frac{v^2}{c^2}} \Rightarrow S = \int_0^t dt\cdot c\sqrt{1-\frac{v^2}{c^2}} \quad t=0 \Rightarrow S=0$$

$$u^2(t=0) = \frac{p^2(t=0)}{mc} = \frac{p_{0y}}{mc}$$

Из (5) $\quad A = \dfrac{p_{0y}}{mc}$

Подст. (5) → (1) и (5) → (2)

$$\frac{du^0}{ds} = \frac{e}{mc^2} E \left(\frac{p_{0y}}{mc} \cdot ch\left(\sqrt{E^2-H^2}\,\frac{e}{mc^2}\, s\right) + B \cdot sh\left(\sqrt{E^2-H^2}\,\frac{e}{mc^2}\, s\right)\right)$$

$$\frac{du^1}{ds} = \frac{e}{mc^2} H \left(\frac{p_{0y}}{mc} \cdot ch\left(\sqrt{E^2-H^2}\,\frac{e}{mc^2}\, s\right) + B \cdot sh\left(\sqrt{E^2-H^2}\,\frac{e}{mc^2}\, s\right)\right)$$

Интегр. по S:

$$\begin{cases} u^0 = \frac{e}{mc^2} E \left(\frac{p_{0y}}{mc} \cdot \frac{\cancel{mc^2}}{\cancel{e}\sqrt{E^2-H^2}}\, sh\left(\sqrt{E^2-H^2}\,\frac{e}{mc^2}\, s\right) + \right. \\ \left. + \frac{B\, mc^2}{e\sqrt{E^2-H^2}}\, ch\left(\sqrt{E^2-H^2}\,\frac{e}{mc^2}\, s\right)\right) + C \\ u^1 = \frac{e}{mc^2} H(\dots) + \mathcal{D} \end{cases}$$

н.у. для u^0 и u^1 $\quad u^0(t=0) = \frac{\mathcal{E}_0}{mc^2}$

$$u^1(t=0) = \frac{p_{0x}}{mc}$$

$$\begin{cases} \frac{\mathcal{E}_0}{mc^2} = \frac{e}{mc^2} E \frac{mc^2 B}{e\sqrt{E^2-H^2}} + C = \frac{EB}{\sqrt{E^2-H^2}} + C \\ \frac{p_{0x}}{mc} = \frac{HB}{\sqrt{E^2-H^2}} + \mathcal{D} \end{cases}$$

подст. u^0 и u^1 (3)

$$\frac{p_{0y}}{mc} \cdot \frac{e}{mc^2} \sqrt{E^2-H^2}\, sh(\dots) - B \cdot \frac{e}{mc^2} \sqrt{E^2-H^2}\, ch(\dots) =$$

$$= \left(\frac{eE}{mc^2}\right)^2 \left(\frac{p_{0y}\, c}{e\sqrt{E^2-H^2}} \cdot sh(\ldots) - \frac{B \cdot mc^2}{e\sqrt{E^2-H^2}}\, ch(\ldots)\right) +$$

$$+\, C \cdot \frac{eE}{mc^2} - \left(\frac{eH}{mc^2}\right)^2 \left(\frac{c\, p_{0y}}{e\sqrt{E^2-H^2}}\, sh(\ldots) + \frac{B \cdot mc^2}{e\sqrt{E^2-H^2}}\, ch(\ldots)\right) - \mathcal{D}\, \frac{eH}{mc^2}$$

незалежн. функц. $\{e^x, e^{-x}, 1\}$ прирівн. між собою

коеф.

$$0 = \frac{e}{mc^2}\left(CE - \mathcal{D}H\right) \Rightarrow C = \frac{H\mathcal{D}}{E} \quad \text{подст. } 6\ (6)$$

$$\begin{cases} \dfrac{EB}{\sqrt{E^2-H^2}} + \dfrac{H}{E}\,\mathcal{D} = \dfrac{\mathcal{E}_0}{mc^2} \\ \mathcal{D} + \dfrac{HB}{\sqrt{E^2-H^2}} = \dfrac{p_{0x}}{mc} \end{cases} \Rightarrow \mathcal{D} = \frac{p_{0x}}{mc} - \frac{H}{\sqrt{E^2-H^2}}\, B \Rightarrow$$

$$\Rightarrow B \cdot \left(\frac{E}{\sqrt{E^2-H^2}} - \frac{H^2}{E\sqrt{E^2-H^2}}\right) = \frac{\mathcal{E}_0}{mc^2} - \frac{H}{E}\,\frac{p_{0x}}{mc}$$

$$B\left(\frac{E^2-H^2}{E\sqrt{E^2-H^2}}\right) = \frac{\mathcal{E}_0}{mc^2} - \frac{H}{E}\,\frac{p_{0x}}{mc} \Rightarrow$$

$$\begin{cases} B = \dfrac{E}{\sqrt{E^2-H^2}\, mc}\left(\dfrac{\mathcal{E}_0}{c} - \dfrac{H}{E}\, p_{0x}\right) \\ \mathcal{D} = \dfrac{p_{0x}}{mc} - \dfrac{H \cdot E}{(E^2-H^2)\, mc}\left(\dfrac{\mathcal{E}_0}{c} - \dfrac{H}{E}\, p_{0x}\right) \\ C = \dfrac{H}{E}\,\dfrac{p_{0x}}{mc} - \dfrac{H^2}{(E^2-H^2)\, mc}\left(\dfrac{\mathcal{E}_0}{c} - \dfrac{H}{E}\, p_{0x}\right) \end{cases} \quad (7)$$

Из (5) и (6)

$$u_2 = \frac{p_{0y}}{mc}\,ch\left(\sqrt{E^2-H^2}\cdot\frac{e}{mc^2}\,s\right) + \frac{E}{\sqrt{E^2-H^2}\,mc}\left(\frac{\varepsilon_0}{c} - \frac{H}{E}\,p_{0x}\right)sh\left(\sqrt{E^2-H^2}\ldots\right)$$

$$= \frac{dx^2}{ds} \Rightarrow x^2 = y = \frac{p_{0y}}{mc}\cdot\frac{mc^2}{e\sqrt{E^2-H^2}}\,sh\left(\sqrt{E^2-H^2}\,\frac{es}{mc^2}\right) +$$

$$+ \frac{E}{\sqrt{E^2-H^2}\,mc}\cdot\frac{mc^2}{e\sqrt{E^2-H^2}}\left(\frac{\varepsilon_0}{c} - \frac{H}{E}\,p_{0x}\right)ch\left(\sqrt{E^2-H^2}\cdot\frac{e}{mc^2}\,s\right) + A_1$$

$$y(t=0) = 0 \qquad y\Big|_t = 0 = \frac{E\cdot c}{(E^2-H^2)e}\left(\frac{\varepsilon_0}{c} - \frac{H}{E}\cdot p_{0x}\right) = -A_1$$

$$y = \frac{p_{0y}\cdot c}{e\sqrt{E^2-H^2}}\,sh\left(\sqrt{E^2-H^2}\,\frac{es}{mc^2}\right) + \frac{cE}{e(E^2-H^2)}\left(\frac{\varepsilon_0 E - Hp_{0x}c}{cE}\right)\cdot$$

$$\cdot\left(ch\left(\sqrt{E^2-H^2}\cdot\frac{e}{mc^2}\,s\right) - 1\right)$$

$$y = \frac{c\,p_{0y}}{e\sqrt{E^2-H^2}}\,sh\left(\frac{e\sqrt{E^2-H^2}\cdot s}{mc^2}\right) + \frac{\varepsilon_0 E - c\,p_{0x}H}{e(E^2-H^2)}\left(ch\left(\frac{e\sqrt{E^2-H^2}\,s}{mc^2}\right) - 1\right)$$

Из (7) нах. выр. u^0 и u^1 x, ct

$$u^1 = H\left(\frac{p_{0y}}{mc\sqrt{E^2-H^2}}\,sh(\ldots) + \frac{E}{(E^2-H^2)mc}\left(\frac{E\cdot\varepsilon_0 - p_{0x}\cdot cH}{c\cdot E}\right)ch(\ldots)\right) + \frac{p_{0x}}{mc} -$$

$$- \frac{H\cdot E}{(E^2-H^2)\,mc}\left(\frac{\varepsilon_0 E - cp_{0x}\cdot H}{cE}\right)$$

$$x = \frac{p_{0y} H}{mc\sqrt{E^2-H^2}} \cdot \frac{mc^2}{e\sqrt{E^2-H^2}}\, ch(\ldots) + \frac{H \cdot E \cdot mc^2}{(E^2-H^2)^{3/2} \cdot \cancel{mc} \cdot e} \cdot \frac{(E\,\mathcal{E}_0 - p_{0x} \cdot cH)}{c \cdot E}\, sh(\ldots) +$$

$$+ \left(\frac{p_{0x}}{mc} - \frac{HE^2 \mathcal{E}_0 - c\, p_{0x} E \cdot H^2}{(E^2-H^2)\, mc^2 E}\right) s + A_r =$$

$$\Rightarrow x\Big|_{t=0} = 0 = \frac{c\, p_{0y} H}{e(E^2-H^2)} + A_r \Big| = \frac{c \cdot p_{0y} H}{e(E^2-H^2)} \left(ch(\ldots) - 1\right) +$$

$$+ \frac{H(\mathcal{E}_0 \cdot E - c \cdot p_{0x} H)}{e(E^2-H^2)^{3/2}}\, sh(\ldots) + \left(\frac{c\, p_{0x}(E^2-\cancel{H^2}) - H \cdot E \cdot \mathcal{E}_0 + \cancel{c \cdot p_{0x} \cdot H^2}}{mc^2(E^2-H^2)}\right) \cdot s$$

$$x = \frac{E(c\, p_{0x} E - H \cdot \mathcal{E}_0)}{mc^2(E^2-H^2)} \cdot s + \frac{H(\mathcal{E}_0 E - c\, p_{0x} H)}{e(E^2-H^2)^{3/2}}\, sh\left(\sqrt{E^2-H^2} \cdot \frac{es}{mc^2}\right) +$$

$$+ \frac{c\, p_{0y} H}{e(E^2-H^2)} \left(ch\left(\frac{\sqrt{E^2-H^2} \cdot e}{mc^2}\, s\right) - 1\right)$$

$$u^0 = E\left(\frac{p_{0x}}{mc\sqrt{E^2-H^2}}\, sh(\ldots) + \frac{E}{(E^2-H^2)mc} \left(\frac{E\,\mathcal{E}_0 - p_{0x} \cdot c \cdot H}{eE}\right) ch(\ldots)\right) +$$

$$+ \frac{p_{0x}}{mc} \frac{H}{E} - \frac{H^2}{(E^2-H^2)mc} \left(\frac{E \cdot \mathcal{E}_0 - c \cdot p_{0x} \cdot H}{eE}\right)$$

$ct(t=0) = 0$ $\qquad \frac{dx^0}{ds} = u^0 \qquad x^0 = ct$

$$ct = \frac{H(p_{0x} \cdot cE - H\,\mathcal{E}_0)}{mc^2(E^2-H^2)} \cdot s + \frac{E(E \cdot \mathcal{E}_0 - c \cdot p_{0x} H)}{e(E^2-H^2)^{3/2}}\, sh\left(\sqrt{E^2-H^2}\, \frac{es}{mc^2}\right) +$$

$$+ s\left(\frac{p_{0x} \cdot Hc(E^2-H^2) - H^2(E \cdot \mathcal{E}_0 - c\, p_{0x} H)}{mc^2 E (E^2-H^2)}\right)$$

$$ct = \frac{H(p_{0x} \cdot cE - H \cdot \mathcal{E}_0)}{mc^2(E^2 - H^2)} \cdot s + \frac{E(E \cdot \mathcal{E}_0 - c p_{0x} \cdot H)}{e(E^2 - H^2)^{3/2}} \cdot sh\left(\sqrt{E^2 - H^2}\,\frac{es}{mc^2}\right) +$$

$$+ \frac{c\, p_{0y} E}{e(E^2 - H^2)}\left(ch \sqrt{E^2 - H^2}\,\frac{es}{mc^2}\right) - 1)$$

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Доза 2

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Доза 3

10) 54?
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